QUESTION 10 (10 marks)

Figure 9 shows points $O\left(0,0,0\right)$, $P\left(1,\sin\theta,\cos\theta\right)$, and $Q\left(\sqrt{2},1,1\right)$. The vector $\boldsymbol{a}=\overrightarrow{OP}$ and the vector $\boldsymbol{b}=\overrightarrow{PQ}$.

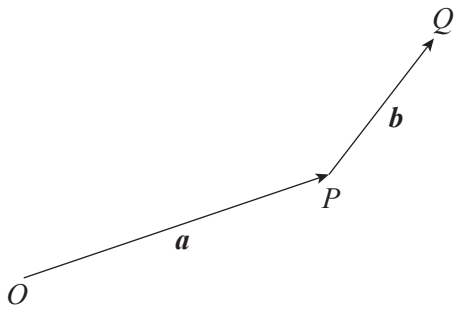

Figure 9

(a) (i) On Figure 9, draw and label the vector $\boldsymbol{a}+\boldsymbol{b}$. (1 mark)

(ii) Calculate $\left|\boldsymbol{a}+\boldsymbol{b}\right|$.

(1 mark)

(b) (i) Show that $\left|\boldsymbol{a}\right|+\left|\boldsymbol{b}\right|=\sqrt{2}+\sqrt{6-2\sqrt{2}-2\left(\sin\theta+\cos\theta\right)}$.

(3 marks)

(ii) State why $2 \leq \sqrt{2} + \sqrt{6 - 2\sqrt{2} - 2(\sin\theta + \cos\theta)}$.

(1 mark)

(c) (i) State the relationship between $\boldsymbol{a}$ and $\boldsymbol{b}$ when $2 = \sqrt{2} + \sqrt{6 - 2\sqrt{2} - 2(\sin\theta + \cos\theta)}$.

(1 mark)

(ii) Hence find an exact value of θ for which $2 = \sqrt{2} + \sqrt{6 - 2\sqrt{2} - 2(\sin\theta + \cos\theta)}$.

(2 marks)

(iii) Hence show that $2 = \sqrt{2} + \sqrt{6 - 4\sqrt{2}}$.

(1 mark)