QUESTION 12 (16 marks)

Consider the system of equations shown below.

$$\begin{cases} x+2y+2z=4 \\ 2x+y-2z=5 \\ 3x+2y-2z=8 \end{cases}$$

(a) (i) Write this system as an augmented matrix.

(1 mark)

(ii) Stating all row operations, show that this system of equations has the following solutions:

$$\begin{aligned} x &= 2+2t \\ y &= 1-2t \\ z &= t \end{aligned}$$

where t is a real parameter.

(3 marks)

(b) Consider three planes in space, P_1, P_2 and P_3, defined by the system of equations shown below.

$$P_1: \quad x + 2y + 2z = 4$$
$$P_2: \quad 2x + y - 2z = 5$$
$$P_3: \; 3x + 2y - 2z = 8$$

(i) Using the information given in part (a)(ii), show that the points $A(2, 1, 0)$ and $B(0, 3, -1)$ are common to all three planes.

(2 marks)

(ii) Show that P_1 and P_2 are perpendicular.

(2 marks)

(c) Figure 12 shows the point $C(0, 6, 2)$ on P_3, the point $D(12, -4, 0)$ on P_1, and the line l through C and D intersecting P_2 at the point E.

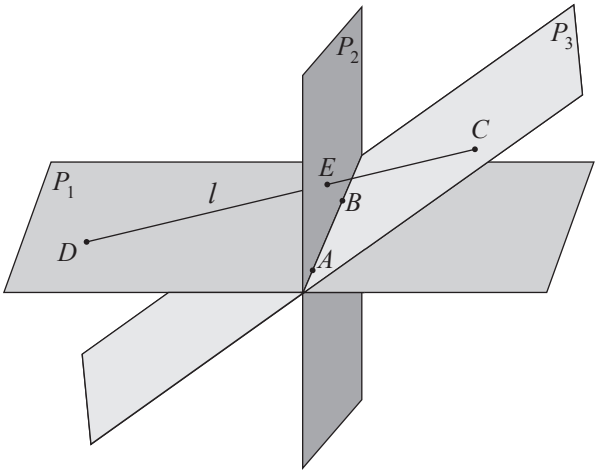

Figure 12

(i) Find, in parametric form, the equation of l.

(2 marks)

(ii) Find the coordinates of E.

(2 marks)

(iii) Find the distance from E to P_1.

(2 marks)

(d) The equations of P_1 and P_3 are used to model two hillsides that meet at a river, as shown in Figure 13.

$$P_1: \quad x + 2y + 2z = 4$$
$$P_3: 3x + 2y - 2z = 8$$

The river is modelled by the line where the two planes meet. A straight bridge, modelled by l, connects $C(0, 6, 2)$ to $D(12, -4, 0)$.

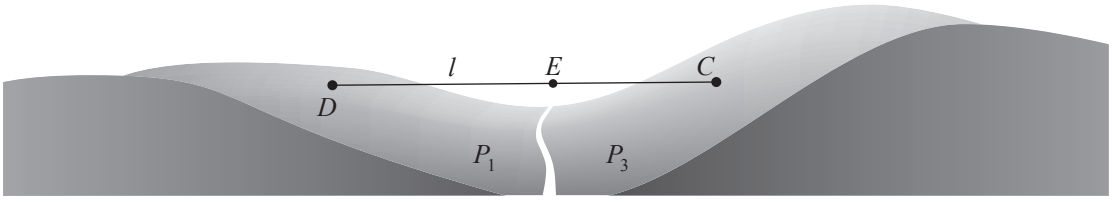

Figure 13

The point E, on the bridge, must be at least 1 unit from P_1 and at least 1 unit from P_3.

Does the model satisfy this condition? Show your calculations.

(2 marks)