Question 4 (9 marks)

Consider three planes in space that are defined by the following system of equations:

$$\begin{aligned} P_1: &\quad x-2y+z=4 \\ P_2: &\quad x+3y-z=0 \\ P_3: &\quad 2x-y+az=a \end{aligned}$$

where a is a real constant.

(a) Write the system of equations in augmented matrix form.

(1 mark)

(b) Clearly stating all row operations, show that the augmented matrix reduces to

$$\begin{bmatrix} 1 & -2 & 1 & : & 4 \\ 0 & 5 & -2 & : & -4 \\ 0 & 0 & (4-5a) & : & (28-5a) \end{bmatrix}.$$

(3 marks)

(c) (i) Solve the system of equations for $a = 0$.

(2 marks)

(ii) Solve the system of equations for $a = \frac{4}{5}$.

(1 mark)

(iii) Which *one* of the following figures best represents the configuration of the three planes when $a = \frac{4}{5}$? Justify your answer.

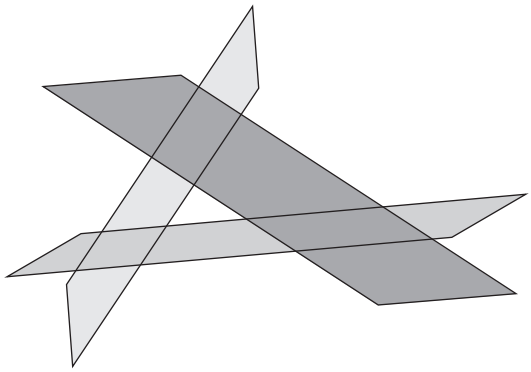

Figure 2

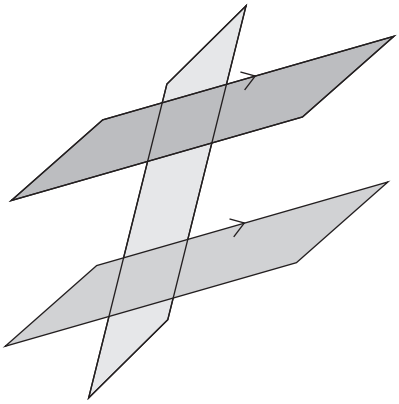

Figure 3

(2 marks)