PART 2 (Questions 11 to 15)

(75 marks)

Question 11 (15 marks)

(a) Calculate the vector (cross) product $[1,-1,-1]\times[1,0,1]$.

(1 mark)

(b) Consider the planes P_1 and P_2 that are defined by the following equations:

$$P_1: x-y-z=4$$
$$P_2: \quad x+z=9.$$

Figure 8 shows P_1, P_2, and the line l_1, where P_1 and P_2 intersect.

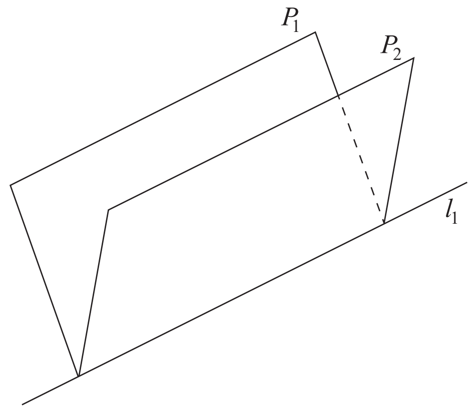

Figure 8

(i) Show that the point $X(9,5,0)$ is on both planes.

(1 mark)

(ii) Hence or otherwise, show that l_1 has the following parametric equations:

$$\begin{cases} x = 9 - t \\ y = 5 - 2t \\ z = t \end{cases} \quad \text{where } t \text{ is real.}$$

(2 marks)

(c) Consider the line l_2, which has the following parametric equations:

$$\begin{cases} x = 3 + 3s \\ y = -s \\ z = 3 \end{cases} \quad \text{where } s \text{ is real.}$$

(i) (1) Show that l_2 intersects l_1.

(2 marks)

(2) Find Y, the point where l_1 and l_2 intersect.

(1 mark)

The line l_2 lies on the plane P_3.

Plane P_3 intersects P_1 and P_2 along the common line l_1, as shown in Figure 9.

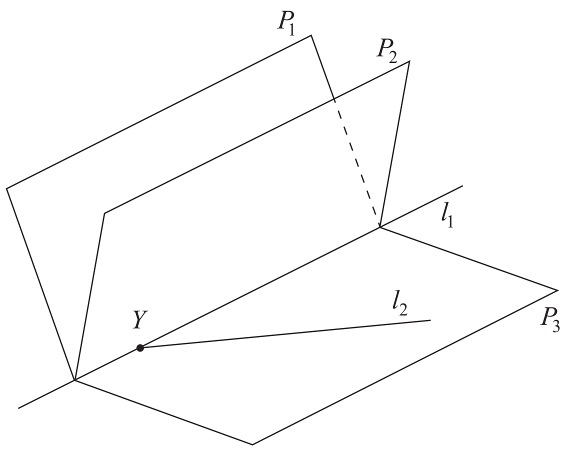

Figure 9

(ii) Show that the equation of P_3 is $x+3y+7z=24$.

(3 marks)

(d) The line l_3 is parallel to l_2, as shown in Figure 10.

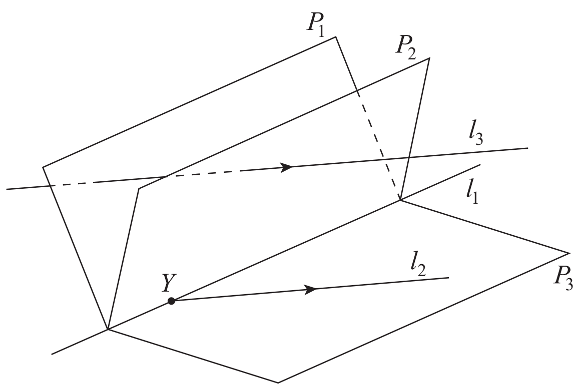

Figure 10

(i) Line l_3 passes through the origin.

Write an equation for l_3.

(1 mark)

(ii) Verify that l_3 does ***not*** lie on P_3.

(1 mark)

(e) Particles are fired from a source located at the origin and travel along l_3.

(i) If the particles travel at a constant speed of $\sqrt{10}$ units/second, show that the particles pass through P_1, 1 second after they have been fired.

(2 marks)

(ii) How many more seconds elapse before the particles pass through P_2?

(1 mark)