Question 6 (9 marks)

(a) Consider the planes that are defined by the following system of equations:

$$P_1 : 3x - y + 2z = 7$$
$$P_2 : \ 2x - y - z = 12.$$

(i) Write this system of equations as an augmented matrix.

(1 mark)

(ii) Clearly stating all row operations, show that there are infinite solutions to this system of equations, and give the solutions in parametric form.

(3 marks)

(iii) Interpret your answer to part (a)(ii) geometrically.

(1 mark)

(b) A third plane is added to the system of equations:

$$P_3 : x - y - (k+2)z = 17.$$

For the system of three equations:

(i) find the value of k for which there are infinite solutions.

(2 marks)

(ii) find the solution for all other values of k.

(1 mark)

(iii) interpret your answer to part (b)(ii) geometrically.

(1 mark)