Question 8 (15 marks)

(a) Consider the planes P_1 and P_2 that are defined by the equations below.

$$P_1: \; x + y + 2z = 4$$
$$P_2: \; 2x - y + z = 8$$

(i) Show that P_1 and P_2 intersect at line l_1, which has the following parametric equations:

$$\begin{cases} x = 4 - t \\ y = -t \\ z = t \end{cases} \quad \text{where } t \text{ is a real parameter.}$$

Clearly state all row operations.

(3 marks)

(ii) Show that the points $A(0, -4, 4)$ and $B(1, -3, 3)$ are on l_1.

(2 marks)

(iii) Show that the point $C(4, 2, 2)$ is on P_2.

(1 mark)

(b) Figure 6 shows P_1 and P_2, and the line l_1 where P_1 and P_2 intersect.

The normal to P_2 through C meets P_1 at the point D.

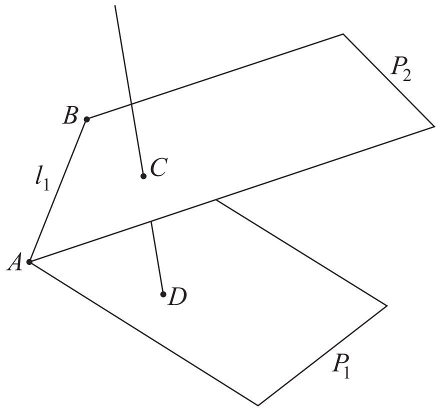

Figure 6

(i) Find the equation of the normal to P_2 through C.

(2 marks)

(ii) Show that D has coordinates $(0, 4, 0)$.

(2 marks)

(iii) From part (a)(i), the parametric equations for l_1 are:

$$\begin{cases} x = 4 - t \\ y = -t \\ z = t \end{cases} \quad \text{where } t \text{ is a real parameter.}$$

Find the coordinates of the point on l_1 that is closest to $D(0, 4, 0)$.

(3 marks)

(iv) How much closer is D to P_2 than it is to l_1?

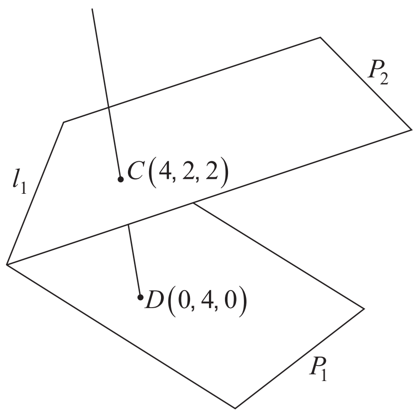

Figure 7

(2 marks)