Question 8 (15 marks)

Points $A(5,-1,-3)$, $B(5,-3,-1)$, and $D(1,-1,1)$ are on the circumference of a circle with centre $C(3,-1,-1)$ on the plane P_1, as shown in Figure 8.

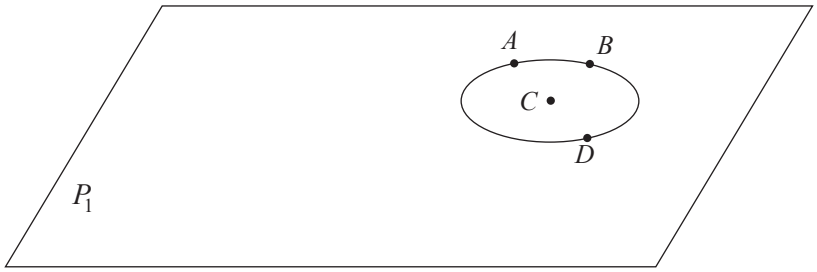

Figure 8

(a) (i) Find $\overrightarrow{BA} \times \overrightarrow{BD}$.

(2 marks)

(ii) Hence show that the equation of plane P_1 is $x + y + z = 1$.

(2 marks)

(b) (i) Show that AD is a diameter of the circle.

(1 mark)

(ii) Find the radius of the circle.

(1 mark)

(c) Point $E(8, -4, -3)$ is on the plane P_1.

Show that the parametric equations of the line through E and B are:

$$\begin{cases} x = 8 - 3t \\ y = -4 + t \\ z = -3 + 2t \end{cases}$$ where t is a real parameter.

(2 marks)

(d) The equation of the circle on P_1 with centre C and passing through A, B, and D is:

$$(x-3)^2 + (y+1)^2 + (z+1)^2 = 8.$$

Show that the line through E and B intersects the circle again at $X\left(\frac{11}{7}, -\frac{13}{7}, \frac{9}{7}\right)$.

(4 marks)

(e) Find the arc length BX.

(3 marks)