## Question 6

(9 marks)
(a) Figure 4 shows the triangle $A B C$, where $\overrightarrow{C A}=\boldsymbol{a}$ and $\overrightarrow{C B}=\boldsymbol{b}$.
Points $P$ and $Q$ lie on $C A$ and $A B$ respectively, such that

$$
\begin{aligned}
& C P=k P A \\
& B Q=k Q A
\end{aligned}
$$

where $k$ is a positive constant.
Write the following vectors in terms of $a$ and $b$.
(i) $\overrightarrow{A B}$

(ii) $\overrightarrow{P A}$

(1 mark)
(b) (i) Using vectors, show that $P Q$ is parallel to $C B$.

(ii) Using vectors, show that the area of triangle $A P Q$ is $\frac{1}{2(k+1)^{2}}|\boldsymbol{a} \times \boldsymbol{b}|$.

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(c) If $|\boldsymbol{a} \times \boldsymbol{b}|=9$ and $k=4$ :
(i) find the area of triangle $A P Q$
$\qquad$
(ii) find the area of quadrilateral $P Q B C$.


