Question 8 (15 marks)

Consider the planes P_1 and P_2 that are defined by the equations below.

$$P_1 : \ 2x + y - z = 1$$
$$P_2 : \ 2x + 3y - z = 7$$

(a) (i) Clearly stating all row operations, show that P_1 and P_2 intersect at l_1, which has the following parametric equations:

$$\begin{cases} x = t \\ y = 3 \\ z = 2 + 2t \end{cases}$$ where t is a real parameter.

(3 marks)

(ii) Show that the points $A\ (0, 3, 2)$ and $B\ (4, 3, 10)$ are on l_1.

(1 mark)

(iii) The plane P_3 is defined by the following equation: $4x + 3y - 2z = 63$.

Show that l_1 is parallel to P_3.

(2 marks)

(b) From part (a)(iii), the equation for P_3 is: $4x + 3y - 2z = 63$.
Point $Q\ (10, 9, 2)$ is on P_3.

(i) The line l_2 is normal to P_3 through Q.
Find the equation of l_2.

(2 marks)

(ii) Show that l_2 meets l_1 at C, where C is the midpoint of AB.

(3 marks)

(iii) Find the distance from l_1 to P_3.

(2 marks)

The line l_2 meets the plane $P_4: 4x + 3y - 2z = -63$ at T, as shown in Figure 6.

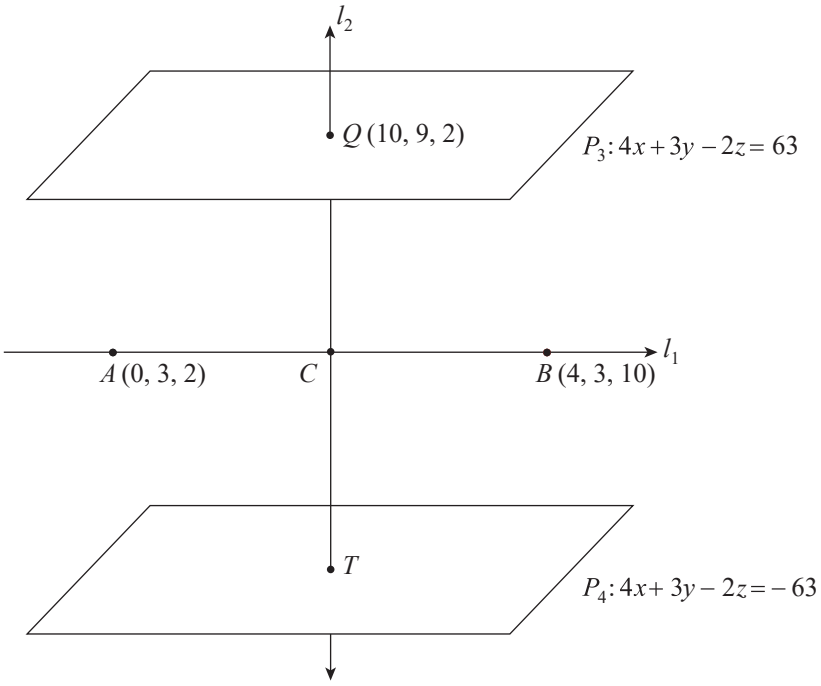

Figure 6

(c) Tick the appropriate box to complete the following statement:

The area of triangle ABT is

- ☐ less than the area of triangle ABQ.
- ☐ the same as the area of triangle ABQ.
- ☐ greater than the area of triangle ABQ.

Justify your answer.

(2 marks)