Question 5   (6 marks)

Consider the following system of equations where m is a non-zero real number.

$$x + y = 0$$
$$mx + z = m^2 - 1$$
$$mx + 2my + (3 - m^2)z = 0$$

(a) Write this system of equations as an augmented matrix.

$$\begin{bmatrix} 1 & 1 & 0 & | & 0 \\ m & 0 & 1 & | & m^2-1 \\ m & 2m & 3-m^2 & | & 0 \end{bmatrix}$$

(1 mark)

(b) Using clearly stated row operations, show that the system in part (a) reduces to:

$$\begin{bmatrix} 1 & 1 & 0 & : & 0 \\ 0 & m & -1 & : & (1-m^2) \\ 0 & 0 & (m^2-4) & : & (1-m^2) \end{bmatrix}.$$

$$\sim \begin{bmatrix} 1 & 1 & 0 & | & 0 \\ 0 & m & -1 & | & 1-m^2 \\ 0 & -m & m^2-3 & | & 0 \end{bmatrix} \begin{matrix} \\ mR_1 - R_2 \\ mR_1 - R_3 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 0 & | & 0 \\ 0 & m & -1 & | & 1-m^2 \\ 0 & 0 & m^2-4 & | & 1-m^2 \end{bmatrix} \begin{matrix} \\ \\ R_2 + R_3 \end{matrix}$$

(c) (i) State a value of m for which there is a unique solution.

$m \neq \pm 2$ and $m \neq 0$

(1 mark)

(ii) Which figure below best represents the solution to this system for $m = -2$?
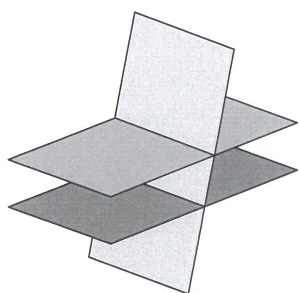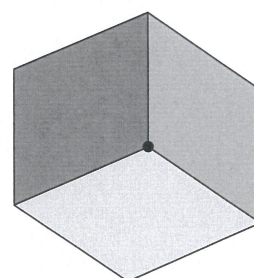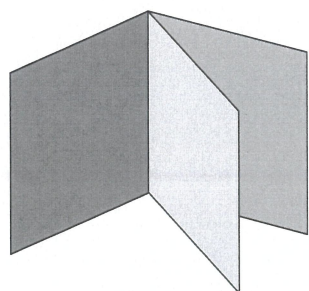
D

(1 mark)

Figure A

Figure B
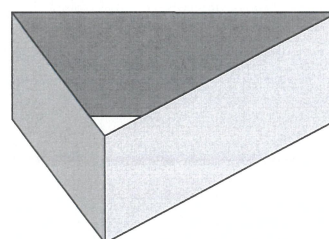
Figure C

Figure D

$m \neq \pm 2$ and $m \neq 0$