Question 6 (9 marks)

(a) Figure 4 shows the triangle ABC, where $\overrightarrow{CA} = \boldsymbol{a}$ and $\overrightarrow{CB} = \boldsymbol{b}$.

Points P and Q lie on CA and AB respectively, such that

$$CP = kPA$$
$$BQ = kQA$$

where k is a positive constant.

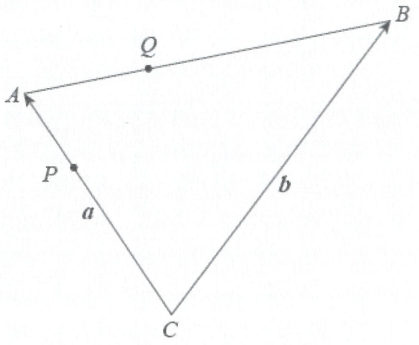

Figure 4

Write the following vectors in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

(i) $\overrightarrow{AB}$

$\underline{b} - \underline{a}$

(1 mark)

(ii) $\overrightarrow{PA}$

$\frac{1}{k+1}\underline{a}$

(1 mark)

(b) (i) Using vectors, show that PQ is parallel to CB.

$\overrightarrow{PQ} = \overrightarrow{PA} + \overrightarrow{AQ}$

$= \frac{1}{k+1}\underline{a} + \frac{1}{k+1}(\underline{b} - \underline{a})$

$= \frac{1}{k+1}\underline{b}$

$\overrightarrow{PQ} = \frac{1}{k+1}\overrightarrow{CB}$

$\therefore$ PQ is parallel to CB

(2 marks)

(ii) Using vectors, show that the area of triangle APQ is $\frac{1}{2(k+1)^2}|\boldsymbol{a}\times\boldsymbol{b}|$.

Area $\Delta APQ = \frac{1}{2}\left|\overrightarrow{PA}\times\overrightarrow{PQ}\right|$

$= \frac{1}{2}\left|\frac{1}{k+1}\underline{a}\times\frac{1}{k+1}\underline{b}\right|$

$= \frac{1}{2(k+1)^2}|\underline{a}\times\underline{b}|$

(2 marks)

(c) If $|\boldsymbol{a}\times\boldsymbol{b}|=9$ and $k=4$:

(i) find the area of triangle APQ

Area $\Delta APQ = \frac{9}{50}$ units^2

(1 mark)

(ii) find the area of quadrilateral $PQBC$.

Area $\Delta ABC = \frac{9}{2}$ units^2

$\therefore$ Area $PQBC = \frac{9}{2} - \frac{9}{50}$

$= \frac{108}{25}$ units^2

(2 marks)