Question 8 (15 marks)

Consider the planes P_1 and P_2 that are defined by the equations below.

$$P_1 : 2x + y - z = 1$$
$$P_2 : 2x + 3y - z = 7$$

(a) (i) Clearly stating all row operations, show that P_1 and P_2 intersect at l_1, which has the following parametric equations:

$$\begin{cases} x = t \\ y = 3 \\ z = 2 + 2t \end{cases} \quad \text{where } t \text{ is a real parameter.}$$

$$\left[\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 2 & 3 & -1 & 7 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 0 & 2 & 0 & 6 \end{array}\right] \begin{array}{l} \\ R_2 - R_1 \end{array}$$

Let $x = t$

$R_2 \Rightarrow y = 3$

$R_1 \Rightarrow 2t + 3 - z = 1$

$z = 2 + 2t$

(3 marks)

(ii) Show that the points A (0, 3, 2) and B (4, 3, 10) are on l_1.

$t = 0 \Rightarrow A(0, 3, 2)$

$t = 4 \Rightarrow B(4, 3, 10)$

(1 mark)

(iii) The plane P_3 is defined by the following equation: $4x + 3y - 2z = 63$.

Show that l_1 is parallel to P_3.

$[4, 3, -2]$ is normal to P_3

$[1, 0, 2]$ is parallel to l_1

$[4, 3, -2] \bullet [1, 0, 2] = 0 \quad \therefore l_1$ is parallel to P_3

(2 marks)

(b) From part (a)(iii), the equation for P_3 is: $4x + 3y - 2z = 63$.

Point Q (10, 9, 2) is on P_3.

(i) The line l_2 is normal to P_3 through Q.

Find the equation of l_2.

The equation of l_2 is $\underset{\sim}{r} = [10, 9, 2] + \lambda[4, 3, -2]$

ie. $\begin{cases} x = 10 + 4\lambda \\ y = 9 + 3\lambda \\ z = 2 - 2\lambda \end{cases}$

(2 marks)

(ii) Show that l_2 meets l_1 at C, where C is the midpoint of AB.

$\begin{cases} 10 + 4\lambda = t \\ 9 + 3\lambda = 3 \\ 2 - 2\lambda = 2 + 2t \end{cases}$

$9 + 3\lambda = 3$

$3\lambda = -6$

$\lambda = -2$

$\lambda = -2 \Rightarrow C(2, 3, 6)$, the midpoint of AB

{Check} $t = 2 \Rightarrow C(2, 3, 6)$ ✓ consistent

(3 marks)

(iii) Find the distance from l_1 to P_3.

$\overrightarrow{CQ} = [8, 6, -4]$

$|\overrightarrow{CQ}| = \sqrt{8^2 + 6^2 + 4^2}$

$= \sqrt{116}$ units

(2 marks)

The line l_2 meets the plane $P_4: 4x+3y-2z=-63$ at T, as shown in Figure 6.

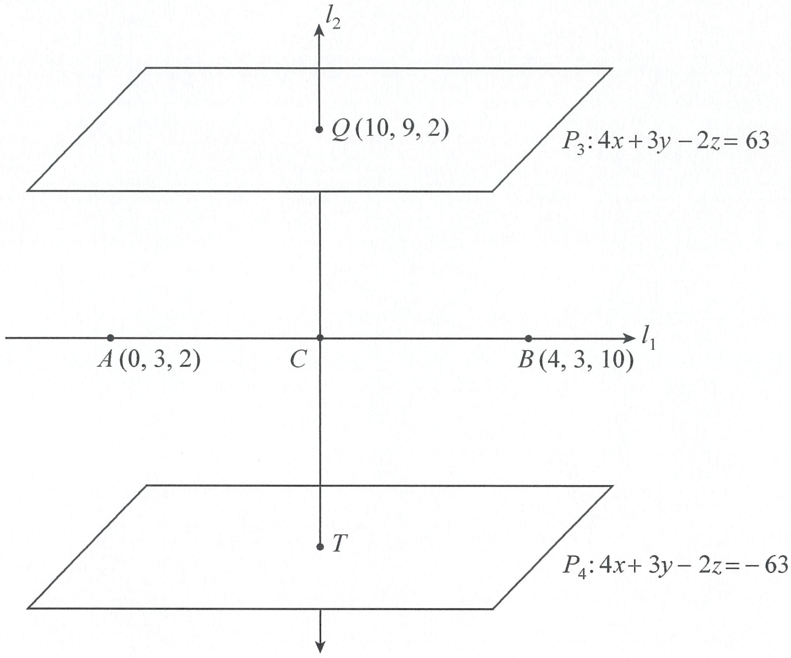

Figure 6

(c) Tick the appropriate box to complete the following statement:

The area of triangle ABT is

☐ less than the area of triangle ABQ.

☐ the same as the area of triangle ABQ.

☑ greater than the area of triangle ABQ.

Justify your answer.

P_3 and P_4 are equidistant from the origin

$\therefore$ The distance between P_3 and P_4 is $\frac{126}{\sqrt{29}}$ units (23.4 units)

From (b)(iii) the distance from C to Q is $\sqrt{116}$ units (10.8 units)

$\therefore$ C is closer to Q than T

$\therefore$ Area $\Delta ABT > \Delta ABQ$

(2 marks)