Question 3 (8 marks)

Figure 2 shows the quadrilateral $ABCD$, where $\overrightarrow{AB} = \boldsymbol{a}$, $\overrightarrow{BC} = \boldsymbol{b}$, and $\overrightarrow{CD} = \boldsymbol{c}$.

The points E, F, G, and H are the midpoints of the sides AB, BC, CD, and DA respectively.

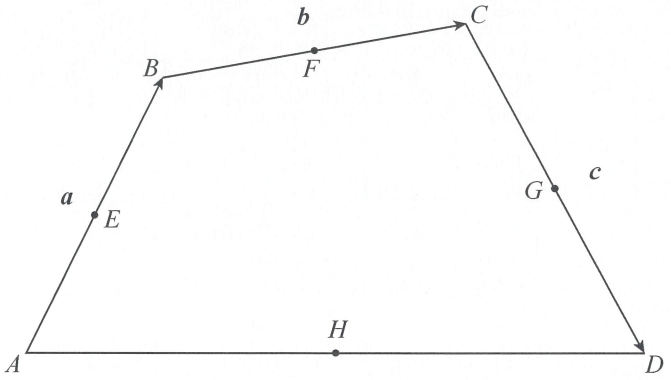

Figure 2

(a) Find the following vectors in terms of $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$.

(i) $\overrightarrow{AD}$

$\overrightarrow{AD} = \underline{a} + \underline{b} + \underline{c}$

(1 mark)

(ii) $\overrightarrow{EF}$

$\overrightarrow{EF} = \frac{1}{2}\underline{a} + \frac{1}{2}\underline{b}$

$= \frac{1}{2}(\underline{a} + \underline{b})$

(1 mark)

(iii) $\overrightarrow{HG}$

$\overrightarrow{HG} = \frac{1}{2}(\underline{a} + \underline{b} + \underline{c}) - \frac{1}{2}\underline{c}$

$= \frac{1}{2}(\underline{a} + \underline{b})$

(2 marks)

(b) (i) Explain why $EFGH$ is a parallelogram.

$\overrightarrow{EF} = \overrightarrow{HG}$

$\therefore$ EF and HG are parallel and equal in length

$\therefore$ EFGH is a parallelogram

(2 marks)

(ii) Show that the area of $EFGH$ is $\frac{1}{4}\left|(\boldsymbol{a}\times\boldsymbol{b})+(\boldsymbol{a}\times\boldsymbol{c})+(\boldsymbol{b}\times\boldsymbol{c})\right|$.

Area of EFGH $= \left|\overrightarrow{EF} \times \overrightarrow{FG}\right|$

$$= \left|\frac{1}{2}(\underline{a}+\underline{b}) \times \frac{1}{2}(\underline{b}+\underline{c})\right|$$

$$= \frac{1}{4}\left|\underline{a}\times\underline{b} + \underline{a}\times\underline{c} + \underline{b}\times\underline{b} + \underline{b}\times\underline{c}\right| \quad (\underline{b}\times\underline{b} = 0)$$

$$= \frac{1}{4}\left|\underline{a}\times\underline{b} + \underline{a}\times\underline{c} + \underline{b}\times\underline{c}\right|$$

(2 marks)