## Question 4 (9 marks)

The points $A(1,0,4)$, $B(5,4,0)$, and $C(7,-6,10)$ form the triangle $ABC$, as shown in Figure 2.

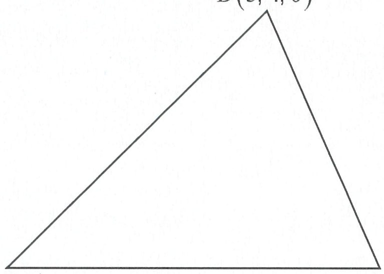


**Figure 2**

(a) (i) Find $\overrightarrow{AB}$.

$\overrightarrow{AB} = [4, 4, -4]$

(1 mark)

(ii) Find $\overrightarrow{AB} \times \overrightarrow{AC}$.

$\overrightarrow{AC} = [6, -6, 6]$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ 4 & 4 & -4 \\ 6 & -6 & 6 \end{vmatrix} = [0, -48, -48]$$

(2 marks)

(iii) Find the ***exact*** area of triangle $ABC$.

$$\text{Area } \Delta ABC = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$$

$$= \frac{48}{2}\sqrt{2}$$

$$= 24\sqrt{2} \text{ units}^2$$

(2 marks)

(b) The point $M(4,3,1)$ divides $AB$ internally in the ratio $3:1$. The point $N(5,-4,8)$ divides $AC$ internally in the ratio $2:1$. That is, $\overrightarrow{AM}=3\overrightarrow{MB}$ and $\overrightarrow{AN}=2\overrightarrow{NC}$, as shown in Figure 3.

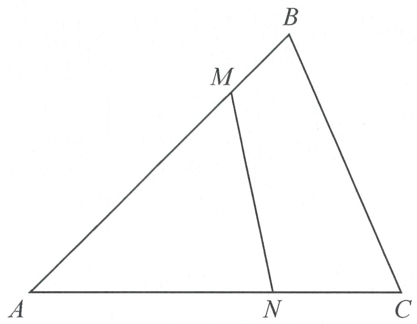


**Figure 3**

(i) Find $\overrightarrow{AM}$ in terms of $\overrightarrow{AB}$.

$\overrightarrow{AM} = \frac{3}{4}\overrightarrow{AB} = [3, 3, -3]$

(1 mark)

(ii) Find the ***exact*** area of triangle $AMN$.

$\overrightarrow{AN} = \frac{2}{3}\overrightarrow{AC}$

Area $\Delta AMN = \frac{1}{2}\left|\overrightarrow{AM}\times\overrightarrow{AN}\right| = \frac{1}{\cancel{2}}\times\frac{\cancel{3}}{4}\times\frac{\cancel{2}}{\cancel{3}}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right| = 12\sqrt{2}$ units$^2$

(1 mark)

(iii) Find the coordinates of a point, $P$, on $AC$, such that:

the area of triangle $AMP = \frac{1}{12}$ the area of triangle $ABC$.

Let $\overrightarrow{AP} = k.\overrightarrow{AC}$

Area $\Delta AMP = \frac{1}{2}\left|\overrightarrow{AM}\times\overrightarrow{AP}\right| = \frac{1}{2}\times\frac{3}{4}\times k\left|\overrightarrow{AB}\times\overrightarrow{AC}\right| = \frac{3k}{4}\times$ Area $\Delta ABC$

$\therefore\ \frac{3k}{4} = \frac{1}{12}$

$k = \frac{1}{9}$

$\therefore$ Coordinates of P are $\left(\frac{5}{3}, \frac{-2}{3}, \frac{14}{3}\right)$

(2 marks)