Question 6 (9 marks)

(a) Consider the planes that are defined by the following system of equations:

$$P_1 : 3x - y + 2z = 7$$
$$P_2 : 2x - y - z = 12.$$

(i) Write this system of equations as an augmented matrix.

$$\left[\begin{array}{ccc|c} 3 & -1 & 2 & 7 \\ 2 & -1 & -1 & 12 \end{array}\right]$$

(1 mark)

(ii) Clearly stating all row operations, show that there are infinite solutions to this system of equations, and give the solutions in parametric form.

$$\sim \left[\begin{array}{ccc|c} 3 & -1 & 2 & 7 \\ 0 & 1 & 7 & -22 \end{array}\right] \; 2R_1 - 3R_2$$

which has an infinite number of solutions

Let $z = t$

$R_2 \Rightarrow y + 7t = -22$

$y = -7t - 22$

$R_1 \Rightarrow 3x + 7t + 22 + 2t = 7$

$3x = -9t - 15$

$x = -3t - 5$

(3 marks)

(iii) Interpret your answer to part (a)(ii) geometrically.

The two plane intersect in the line $\begin{cases} x = -3t - 5 \\ y = -7t - 22 \\ z = t \end{cases}$

(1 mark)

(b) A third plane is added to the system of equations:

$$P_3 : x - y - (k+2)z = 17.$$

For the system of three equations:

(i) find the value of k for which there are infinite solutions.

Substituting the parametric equations of the line into the equation of P_3 gives

$(-3t-5) - (-7t-22) - (k+2)t = 17$

$-3t - 5 + 7t + 22 - kt - 2t = 17$

$2t - kt = 0$

$(2-k)t = 0$

$\therefore$ when $k=2$ there are an infinite number of solutions

(2 marks)

(ii) find the solution for all other values of k.

$k \neq t \Rightarrow t = 0$

$t=0 \Rightarrow x=-5, \; y=-22, \; z=0$

(1 mark)

(iii) interpret your answer to part (b)(ii) geometrically.

The three planes intersect at the point $(-5, -22, 0)$

(1 mark)