QUESTION 3 (6 marks)

Figure 1 shows rhombus $OPQR$ with $\overrightarrow{OP} = \boldsymbol{p}$ and $\overrightarrow{OR} = \boldsymbol{r}$.

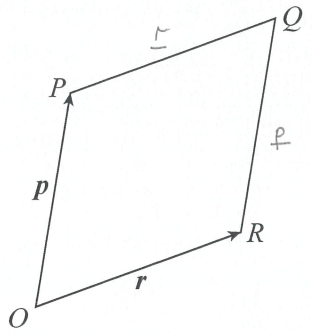

Figure 1

(a) (i) Find $\overrightarrow{OQ}$ in terms of $\boldsymbol{p}$ and $\boldsymbol{r}$.

$\overrightarrow{OQ} = r + p$

(1 mark)

(ii) Find $\overrightarrow{PR}$ in terms of $\boldsymbol{p}$ and $\boldsymbol{r}$.

$\overrightarrow{PR} = r - p$

(1 mark)

(b) (i) Show that $\overrightarrow{OQ} \cdot \overrightarrow{PR} = |\boldsymbol{r}|^2 - |\boldsymbol{p}|^2$.

$\overrightarrow{OQ} \cdot \overrightarrow{PR} = (\underline{r} + \underline{p}) \cdot (\underline{r} - \underline{p})$

$= \underline{r} \cdot \underline{r} - \underline{p} \cdot \underline{p}$

$= |\underline{r}|^2 - |\underline{p}|^2$

(2 marks)

(ii) Hence prove that the diagonals of the rhombus $OPQR$ are perpendicular, giving reasons.

$|\underline{p}| = |\underline{r}|$ since OPQR is a rhombus

$\therefore \overrightarrow{OQ} \cdot \overrightarrow{PR} = 0$

$\therefore \overrightarrow{OQ} \perp \overrightarrow{PR}$

(2 marks)