PART 2 (Questions 11 to 15)

(75 marks)

Question 11 (15 marks)

The points $A(3,1,-1)$, $B(0,2,10)$, and $C(0,0,6)$ are on the plane P.

(a) (i) Find $\overrightarrow{AB}$.

$\overrightarrow{AB} = [-3, 1, 11]$

(1 mark)

(ii) Find $\overrightarrow{AB} \times \overrightarrow{AC}$.

$\overrightarrow{AC} = [-3, -1, 7]$ $\therefore \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ -3 & 1 & 11 \\ -3 & -1 & 7 \end{vmatrix} = [18, -12, 6]$

(1 mark)

(iii) Show that the equation of P is $3x - 2y + z = 6$.

$\frac{1}{6} \overrightarrow{AB} \times \overrightarrow{AC} = [3, -2, 1]$ is normal to the P and $C(0,0,6)$ is a point on P

$\therefore$ The equation of P is $3x - 2y + z = 3(0) - 2(0) + (6)$

$3x - 2y + z = 6$

(2 marks)

(b) (i) Find the equation of the normal to P through the point $D(6,-1,0)$.

$\underset{\sim}{r} = [6, -1, 0] + \lambda [3, -2, 1]$ ie. $\begin{cases} x = 6 + 3\lambda \\ y = -1 - 2\lambda \\ z = \lambda \end{cases}$

(2 marks)

(ii) Show that the normal found in part (b)(i) intersects P at $A(3,1,-1)$.

Substituting the parametric equations above into the equation of P gives

$3(6+3\lambda)-2(-1-2\lambda)+(\lambda)=6$

$18+9\lambda+2+4\lambda+\lambda=6$

$14\lambda=-14$

$\lambda=-1 \Rightarrow x=3,\ y=1,\ z=-1,$ the coordinates of A

(2 marks)

(c) Figure 11 shows the point $E(3,0,11)$ on the normal to P through $B(0,2,10)$.

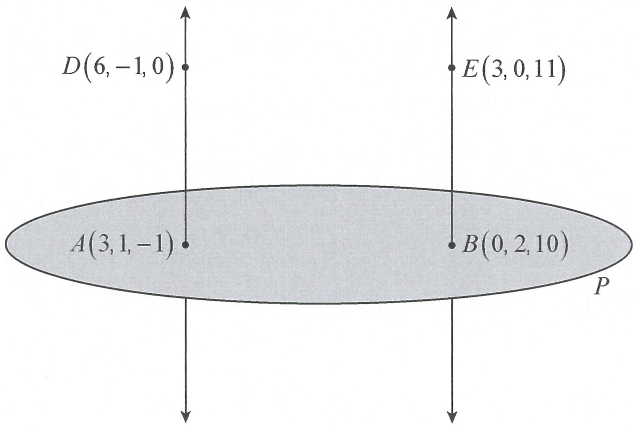

Figure 11

(i) Show that the line through D and E is parallel to P.

$\overrightarrow{DE}=[-3,1,11]$

$[3,-2,1]$ is normal to P

$\overrightarrow{DE}\cdot[3,-2,1]=-9-2+11=0$

$\therefore$ The line through D and E is parallel to P

(2 marks)

(ii) Find the distance from this line to P.

$\overrightarrow{AD} = [3, -2, 1]$

$|\overrightarrow{AD}| = \sqrt{9 + 4 + 1}$

$= \sqrt{14}$ units

(1 mark)

(d) (i) Show that the point $F(-3, 2, 5)$ is the same distance from P as the line that passes through D and E is from P.

$$d = \frac{|3(-3) - 2(2) + (5) - 6|}{\sqrt{3^2 + 2^2 + 1^2}}$$

$$= \frac{14}{\sqrt{14}}$$

$= \sqrt{14}$ units

(2 marks)

(ii) Is F on the line through D and E? Explain your answer.

The equation of the line through D and E is $\underset{\sim}{r} = [6, -1, 0] + \lambda[-3, 1, 11]$

ie. $\begin{cases} x = 6 - 3\lambda \\ y = -1 + \lambda \\ z = 11\lambda \end{cases}$

Trying to solve $\begin{cases} 6 - 3\lambda = -3 \\ -1 + \lambda = 2 \\ 11\lambda = 5 \end{cases}$ gives $\lambda = 3$, $\lambda = 3$, $\lambda = 5/11$ (inconsistent)

$\therefore$ F is not on the line through D and E

(2 marks)