Question 5 (9 marks)

(a) Figure 4 shows the points $A(1, 2, -3)$, $B(5, 3, -2)$, $C(6, 7, -3)$, and $D(2, 6, -4)$.

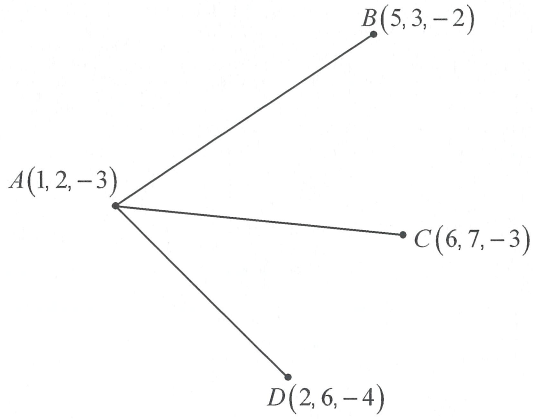

Figure 4

(i) Find $\overrightarrow{AB} \bullet \overrightarrow{AC}$.

$\overrightarrow{AB} = [4, 1, 1]$

$\overrightarrow{AC} = [5, 5, 0]$

$\overrightarrow{AB} \circ \overrightarrow{AC} = 20 + 5 + 0$

$= 25$

(2 marks)

(ii) Find $\cos \angle BAC$.

$\cos(\angle BAC) = \dfrac{25}{\sqrt{18}.\sqrt{50}}$

(1 mark)

(iii) Find $\cos \angle CAD$.

$\overrightarrow{AD} = [1, 4, -1]$ $\quad \overrightarrow{AC} \circ \overrightarrow{AD} = 5 + 20 + 0 = 25$

$\therefore \cos(\angle CAD) = \dfrac{25}{\sqrt{18}.\sqrt{50}}$

(1 mark)

(b) Let $\overrightarrow{OP} = \boldsymbol{p}$ and $\overrightarrow{OQ} = \boldsymbol{q}$.

(i) On Figure 5, clearly show the vector $\overrightarrow{OR} = \boldsymbol{p} + \boldsymbol{q}$.

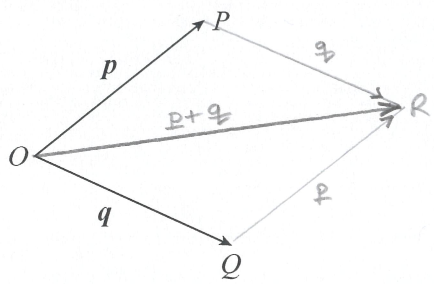

Figure 5 (1 mark)

(ii) If $|\boldsymbol{p}| = |\boldsymbol{q}|$, prove that $\overrightarrow{OR}$ bisects $\angle POQ$.

$$\cos(\angle POR) = \frac{\underset{\sim}{p} \cdot (\underset{\sim}{p} + \underset{\sim}{q})}{|\underset{\sim}{p}| \cdot |\underset{\sim}{p} + \underset{\sim}{q}|} = \frac{|\underset{\sim}{p}|^2 + \underset{\sim}{p} \cdot \underset{\sim}{q}}{|\underset{\sim}{p}| \cdot |\underset{\sim}{p} + \underset{\sim}{q}|}$$

$$\cos(\angle QOR) = \frac{\underset{\sim}{q} \cdot (\underset{\sim}{p} + \underset{\sim}{q})}{|\underset{\sim}{q}| \cdot |\underset{\sim}{p} + \underset{\sim}{q}|} = \frac{|\underset{\sim}{q}|^2 + \underset{\sim}{p} \cdot \underset{\sim}{q}}{|\underset{\sim}{q}| \cdot |\underset{\sim}{p} + \underset{\sim}{q}|}$$

$|\underset{\sim}{p}| = |\underset{\sim}{q}| \Rightarrow \cos(\angle POQ) = \cos(\angle QOR) \quad \therefore \angle POQ = \angle QOR$ (OR bisects $\angle POQ$)

(2 marks)

(c) Figure 6 shows $\overrightarrow{OE} = [2, 5, -7]$ and $\overrightarrow{OF} = [10, 14, 4]$. Find a vector $\overrightarrow{OG}$ that bisects $\angle EOF$.

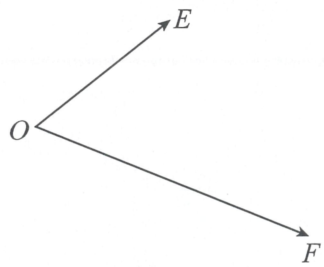

Figure 6

$\frac{1}{2}\overrightarrow{OF} = [5, 7, 2]$

Note that $|\overrightarrow{OE}| = |\overrightarrow{OF}|$

$\therefore \overrightarrow{OG} = \overrightarrow{OE} + \frac{1}{2}\overrightarrow{OF} = [7, 12, -5]$ bisects $\angle EOF$

(2 marks)