**QUESTION 10** (10 marks)

Figure 9 shows points $O(0,0,0)$, $P(1, \sin\theta, \cos\theta)$, and $Q(\sqrt{2}, 1, 1)$. The vector $\boldsymbol{a} = \overrightarrow{OP}$ and the vector $\boldsymbol{b} = \overrightarrow{PQ}$.

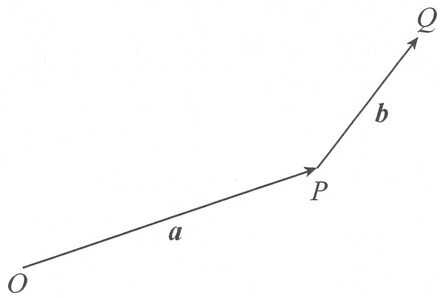


**Figure 9**

(a) (i) On Figure 9, draw and label the vector $\boldsymbol{a} + \boldsymbol{b}$. (1 mark)

(ii) Calculate $|\boldsymbol{a} + \boldsymbol{b}|$.

$|\underline{a} + \underline{b}| = |\overrightarrow{OQ}| = \sqrt{2+1+1}$

$= 2$ units

(1 mark)

(b) (i) Show that $|\boldsymbol{a}| + |\boldsymbol{b}| = \sqrt{2} + \sqrt{6 - 2\sqrt{2} - 2(\sin\theta + \cos\theta)}$.

$|\underline{a}| = \sqrt{1 + \sin^2\theta + \cos^2\theta}$

$= \sqrt{2}$

$|\underline{b}| = \sqrt{(\sqrt{2}-1)^2 + (1-\sin\theta)^2 + (1-\cos\theta)^2}$

$= \sqrt{2 - 2\sqrt{2} + 1 + 1 - 2\sin\theta + \sin^2\theta + 1 - 2\cos\theta + \cos^2\theta}$

$= \sqrt{6 - 2\sqrt{2} - 2(\sin\theta + \cos\theta)}$

$\therefore |\underline{a}| + |\underline{b}| = \sqrt{2} + \sqrt{6 - 2\sqrt{2} - 2(\sin\theta + \cos\theta)}$

(3 marks)

(ii) State why $2 \le \sqrt{2}+\sqrt{6-2\sqrt{2}-2(\sin\theta+\cos\theta)}$.

By the triangle inequality, $|\underset{\sim}{a}+\underset{\sim}{b}| \le |\underset{\sim}{a}|+|\underset{\sim}{b}|$

$\therefore 2 \le \sqrt{2}+\sqrt{6-2\sqrt{2}-2(\sin\theta+\cos\theta)}$

(1 mark)

(c) (i) State the relationship between $\boldsymbol{a}$ and $\boldsymbol{b}$ when $2=\sqrt{2}+\sqrt{6-2\sqrt{2}-2(\sin\theta+\cos\theta)}$.

$\underset{\sim}{a}$ and $\underset{\sim}{b}$ are parallel (and in the same direction)

$\underset{\sim}{b} = k\underset{\sim}{a}$ $(k>0)$

(1 mark)

(ii) Hence find an exact value of $\theta$ for which $2=\sqrt{2}+\sqrt{6-2\sqrt{2}-2(\sin\theta+\cos\theta)}$.

$[\sqrt{2}-1,\ 1-\sin\theta,\ 1-\cos\theta] = k[1,\ \sin\theta,\ \cos\theta]$

$\underset{\sim}{i}$: $\sqrt{2}-1 = k$

$$\left.\begin{aligned}
&\underset{\sim}{j}: && 1-\cancel{\sin\theta} = \sqrt{2}\sin\theta - \cancel{\sin\theta} \\
& && \sin\theta = 1/\sqrt{2} \\
&\underset{\sim}{k}: && 1-\cancel{\cos\theta} = \sqrt{2}\cos\theta - \cancel{\cos\theta} \\
& && \cos\theta = 1/\sqrt{2}
\end{aligned}\right\}\ \theta = \pi/4$$

(2 marks)

(iii) Hence show that $2=\sqrt{2}+\sqrt{6-4\sqrt{2}}$.

$$\begin{aligned}
\theta = \pi/4 \Rightarrow\ 2 &= \sqrt{2}+\sqrt{6-2\sqrt{2}-2.\,2/\sqrt{2}} \\
&= \sqrt{2}+\sqrt{6-2\sqrt{2}-2\sqrt{2}} \\
&= \sqrt{2}+\sqrt{6-4\sqrt{2}}
\end{aligned}$$

(1 mark)

 PLEASE TURN OVER