**Stage 2 Mathematical Methods**

**Differential Calculus Test (Exponential and Logarithmic Functions)**

**Topic 1: Subtopics 1.3, 4.1, 4.2, 4.3**

**Total Marks – 38**

**(Calculator and one A4 page of hand written notes permitted)**

1. (9 marks)

Differentiate the following. There is no need to simplify your answers.

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(3 marks)

1. (12 marks)

Let . Consider the graph of .

1. Find the *exact* values of the axis intercepts of this graph (if they exist).

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(3 marks)

1. Show that the equation of the tangent to the graph at is given by .

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(3 marks)

1. Find the coordinates of the point at which this tangent cuts the y-axis.

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(1 mark)

1. For what values of is defined?

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(1 mark)

1. Show that the graph of is concave upwards for all values of for which it is defined.

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(2 marks)

1. (9 marks)

The graph of is shown below, for

1. Use your graphics calculator to find the coordinates of the maximum visible in the graph above.

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(1 mark)

1. On the axes above, carefully sketch the graph of .

Accurately plot, and label with an , the maximum of this graph.

(2 marks)

1. Use your graphics calculator to complete the following table:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| function |  |  |  |  |
| x-coordinate of maximum |  |  |  |  |

(2 marks)

1. Make a conjecture about the x-coordinate of the maximum of where is a positive integer.

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(1 mark)

1. Prove or disprove the conjecture you made in part (d).

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(3 marks)

1. (8 marks)

A tank initially contains 80 L of liquid. The volume, , of liquid contained inside the tank after minutes is given by where is the original volume of the liquid

1. Show that .

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(1 mark)

1. If the volume is **INCREASING** at a rate of 5 L per minute at the instant the volume reaches 250 L, calculate the value of .

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(2 marks)

1. What volume of liquid is contained within the tank after 10 minutes?

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(1 mark)

1. Calculate the rate at which the volume is changing after 10 minutes?

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(1 mark)

1. *Exactly* how long will it take for the volume to reach 200 L?

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(3 marks)