Question 6 (12 marks)

The concentration of caffeine in the blood plasma of an adult t hours after they have consumed a 150 milligram dose of caffeine can be modelled by the function

$$c(t) = 15\left(e^{-0.3t} - e^{-0.6t}\right) \text{ for } t \geq 0,$$

where the concentration, $c(t)$, is measured in milligrams per litre (mg L^{-1}).

Source: adapted from
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(a) *On the set of axes in Figure 7*, sketch a graph of $y = c(t)$.

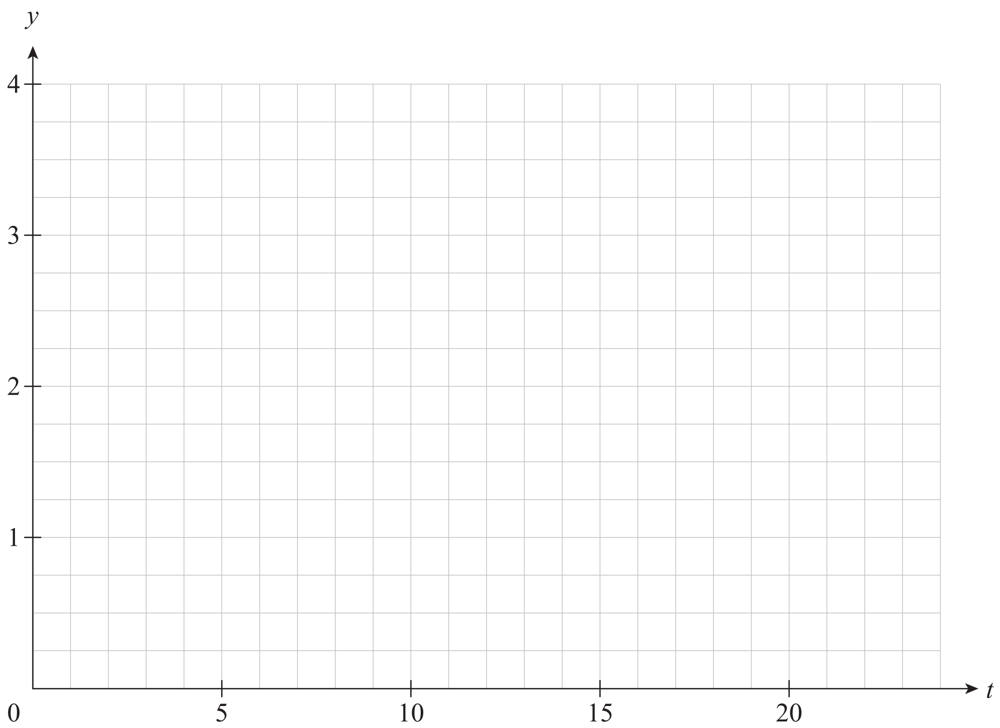

Figure 7

(2 marks)

(b) Using the model above, determine the concentration of caffeine in the blood plasma of an adult 10 hours after they have consumed a 150 milligram dose of caffeine.

(1 mark)

(c) (i) Determine $c'(2)$.

(1 mark)

(ii) Interpret the value of $c'(2)$ in the context of the problem.

(2 marks)

(d) (i) Show that $c'(t) = -4.5e^{-0.3t} + 9e^{-0.6t}$.

(1 mark)

(ii) Hence, by letting $c'(t) = 0$, show that the concentration of caffeine in an adult's blood plasma reaches its maximum at $t = \frac{10}{3}\ln 2$ hours.

(3 marks)

Question 6 continues on page 14.

A general model for the concentration, $c_d(t)$, of caffeine in the blood plasma of an adult t hours after they have consumed a dose of d milligrams of caffeine is

$$c_d(t) = \frac{d}{10}\left(e^{-0.3t} - e^{-0.6t}\right) \text{ for } t \geq 0,$$

where $c_d(t)$ is measured in milligrams per litre (mg L^{-1}).

The maximum concentration of caffeine in an adult's blood plasma in the general model also occurs at $t = \frac{10}{3}\ln 2$ hours.

(e) If the concentration of caffeine in an adult's blood plasma is greater than 15 mg L^{-1}, the adult will experience serious side effects.

Show that the general model predicts that a dose of 600 milligrams of caffeine is the maximum an adult can consume without experiencing serious side effects.

(2 marks)