## Question 7 (9 marks)

The graph of  $y = x \ln x - x$  where x > 0 is shown in Figure 6.



Figure 6

(a)	(i)	Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \ln x$ .																						

(1 mark)

(ii) Hence, using an algebraic approach, determine the *y*-coordinate of the stationary point of  $y = x \ln x - x$ .



(2 marks)

Consider the family of functions of the form  $y = x \ln x - x + (\ln k)x$ , where x > 0 and k is a positive constant. Each function in this family has a stationary point at A, as shown in Figure 7.





(b) Complete the table below by finding the *y*-coordinates of *A* for  $y = x \ln x - x + (\ln k)x$  where k = 1, 2, 3, and 4.

k	Function	y-coordinate of A
1	$x \ln x - x$	
2	$x\ln x - x + (\ln 2)x$	
3	$x\ln x - x + (\ln 3)x$	
4	$x\ln x - x + (\ln 4)x$	

(1 mark)

(c) Hence, make a conjecture for the *y*-coordinate of *A*, for any value of *k*.

(1 mark)

## (d) Prove or disprove your conjecture.

(4 marks)