Question 12 (6 marks)

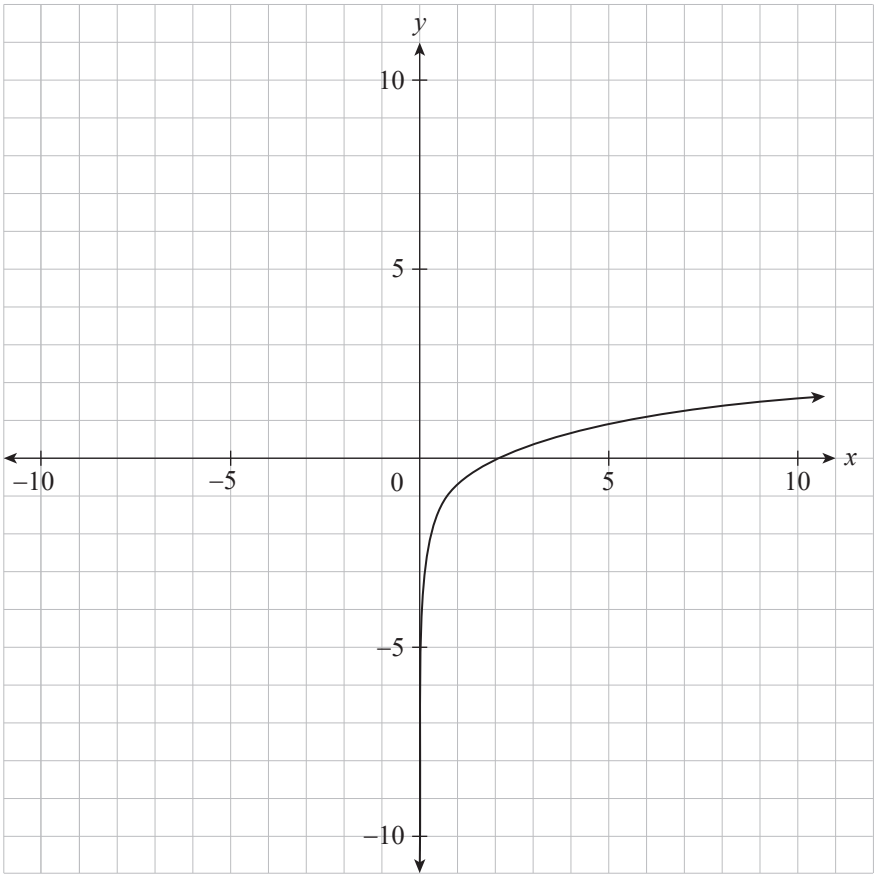
(a) Simplify the following expression:

$\ln\left(\frac{1}{2}x\right)^2 + \ln\left(\frac{1}{2}x\right)$.

Write your answer in the form $a \ln bx$, where a and b are real numbers.

(1 mark)

(b) The graph of the function $y_1 = \ln\left(\frac{1}{2}x\right)$ is shown below.



(i) On the axes above, sketch the graph of the function $y_2 = \ln\left(\frac{1}{2}x\right)^2 + \ln\left(\frac{1}{2}x\right)$. (1 mark)

(ii) Describe the relationship between the graphs of the functions

$y_1 = \ln\left(\frac{1}{2}x\right)$

and

$y_2 = \ln\left(\frac{1}{2}x\right)^2 + \ln\left(\frac{1}{2}x\right).$

(2 marks)

(c) Find the exact value of x such that

$\ln\left(\frac{1}{2}x\right)^2 + \ln\left(\frac{1}{2}x\right) = 3.$

(2 marks)