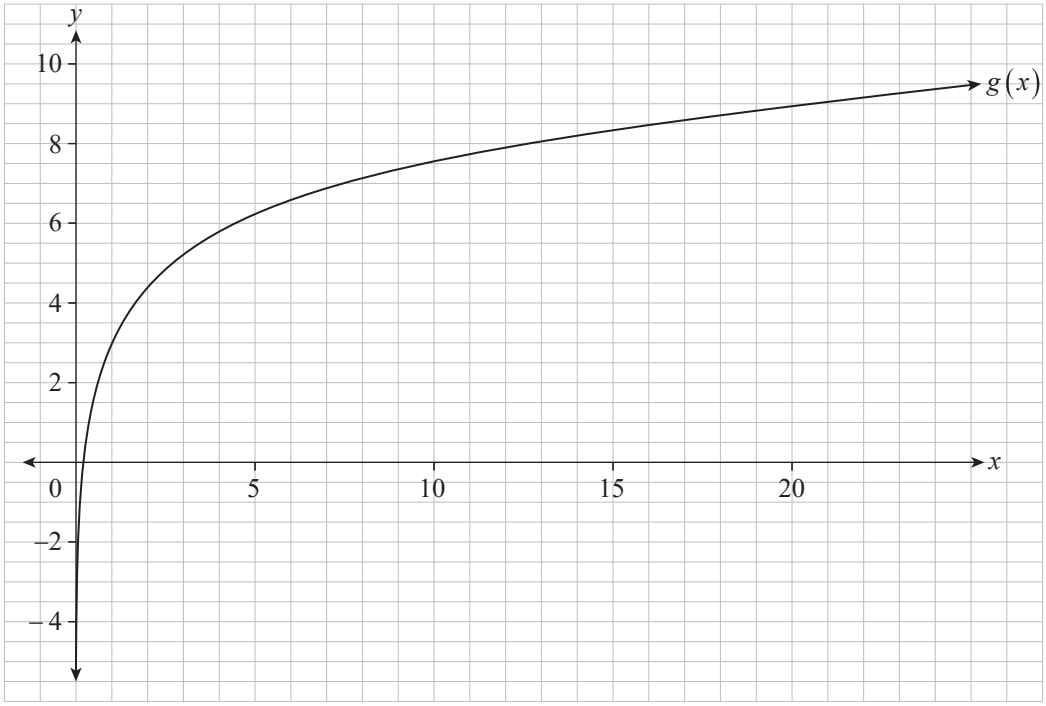
Question 11 (16 marks)

The graph of $y = g(x)$ is shown below, where $g(x) = 2\ln x + 3$ and $x > 0$.



Let $f(x) = (\ln x)^2$.

(a) On the axes above, sketch the curve of $y = f(x)$. Clearly show the coordinates of any intersection points or turning points. (3 marks)

(b) Using algebra, show that the solutions to the equation $f(x) = g(x)$ are $x = \frac{1}{e}$ and $x = e^3$.

(3 marks)

Let $D(x) = g(x) - f(x)$.

(c) (i) Calculate the value of $D(3)$.

(1 mark)

(ii) Interpret your answer to part (c)(i), in relation to the graphs of $y = f(x)$ and $y = g(x)$.

(1 mark)

(iii) State the domain of $D(x)$ if $D(x) \geq 0$.

(1 mark)

(d) (i) Show that $D'(x) = \dfrac{2 - 2\ln x}{x}$.

(1 mark)

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(ii) Determine $D''(x)$.

(2 marks)

(iii) Use $D'(x)$ to calculate the *exact* maximum value of $D(x)$.

(3 marks)

(iv) Use your answer to part (d)(ii) to justify that your answer to part (d)(iii) is the maximum value of $D(x)$.

(1 mark)