## Question $9 \quad$ (11 marks)

Logistic models can be used to understand and plan for change management in a workplace.
One workplace with 100 employees is planning to introduce a new software system called Lisen. Before Lisen is introduced, a small number of employees will undergo training to become skilled at using it. These employees will be 'initially skilled' at using Lisen, and will support others in the workplace to become skilled at using it.

The number of employees, $N$, who are expected to be skilled at using Lisen $t$ weeks after it has been introduced can be modelled by the equation below.

$$
N=\frac{100}{1+9 e^{-0.4 t}}
$$

(a) State the number of employees who are expected to be initially skilled at using Lisen.

(b) State the number of employees who are expected to be skilled at using Lisen 8 weeks after it has been introduced.

(c) Find an expression for $\frac{\mathrm{d} N}{\mathrm{~d} t}$.

(d) Determine when the number of employees who are expected to be skilled at using Lisen is increasing at its greatest rate. Give your answer in weeks, correct to one decimal place.

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The managers of the workplace wish to schedule a training day, on which all employees will be trained in using Lisen.

For the training day, the managers would like the employees to be placed in groups of five, where each group contains at least one employee who is skilled at using Lisen.
The senior manager sets the following target:
'The probability that a randomly selected group of five employees contains at least one employee who is skilled at using Lisen should be at least 0.9'.
(e) A junior manager suggests that the training day can occur once $20 \%$ of employees are skilled at using Lisen.
Show that if the training day occurs when $20 \%$ of employees are skilled at using Lisen, the senior manager's target will not be met.

(2 marks)
(f) (i) Calculate the minimum proportion of employees who would need to be skilled at using Lisen in order for the senior manager's target to be met.

(3 marks)
(ii) Hence, state the minimum number of weeks after the introduction of Lisen that this training day would need to occur.

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