**Question 8** (10 marks)

Figure 9 shows the graph of $y = x\ln\left(x^2 + 2\right)$. The tangent to the graph at $x = 1$ is also shown.

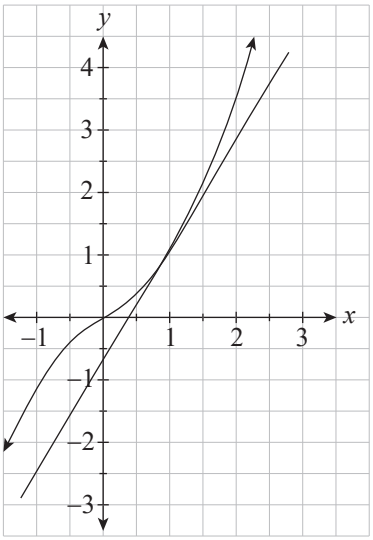


**Figure 9**

(a) (i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \ln\left(x^2 + 2\right) + \dfrac{2x^2}{x^2 + 2}$.

(1 mark)

(ii) Hence show that the tangent to the graph of $y = x\ln\left(x^2 + 2\right)$ at $x = 1$ has equation $y = \left(\ln 3 + \dfrac{2}{3}\right)x - \dfrac{2}{3}$.

(3 marks)

(iii) Determine the $y$-intercept of this tangent.

(1 mark)

Consider the family of functions of the form $y = x\ln\left(x^n + n\right)$ where $n > 0$.

The table below shows the values of the $y$-intercept of the tangent to the graphs of $y = x\ln\left(x^n + n\right)$ at $x = 1$, where $n = 3, 4,$ and $5$.

| $n$ | *Function* | *$y$-intercept of the tangent to the graph of the function at $x = 1$* |
|---|---|---|
| 3 | $y = x\ln\left(x^3 + 3\right)$ | $-\frac{3}{4}$ |
| 4 | $y = x\ln\left(x^4 + 4\right)$ | $-\frac{4}{5}$ |
| 5 | $y = x\ln\left(x^5 + 5\right)$ | $-\frac{5}{6}$ |

(b) Make a conjecture about the value of the $y$-intercept of the tangent to the graph of $y = x\ln\left(x^n + n\right)$ at $x = 1$.

(1 mark)

***Question 8 continues on page 6.***

(c) Prove or disprove the conjecture that you made in part (b) for the $y$-intercept of the tangent to the graph of $y = x\ln\left(x^n + n\right)$ at $x = 1$.

(4 marks)