Question 7 (8 marks)

Consider the function $f(x) = 8 - 2^{0.5x}$. The graph of $y = f(x)$ is shown in Figure 8.

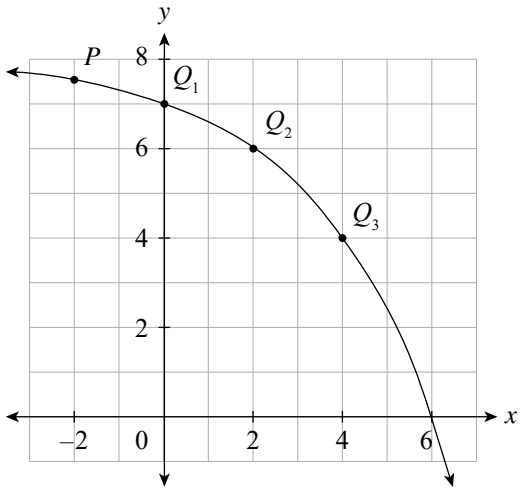

Figure 8

Points P, Q_1, Q_2, and Q_3 are shown on the graph in Figure 8. Their coordinates are: $P\ (-2,\ 7.5)$, $Q_1\ (0,\ 7)$, $Q_2\ (2,\ 6)$, and $Q_3\ (4,\ 4)$.

(a) Chords can be drawn from P to each of Q_1, Q_2, or Q_3.

(i) Identify which of the following chords has a slope that would provide the best approximation for the slope of the tangent to the graph of $y = f(x)$ at P. Tick the appropriate box to indicate your answer.

PQ_1 ☐ PQ_2 ☐ PQ_3 ☐

(1 mark)

(ii) Calculate the slope of the chord that you selected in part (a)(i).

(2 marks)

(b) (i) Show that $f(x)$ can be expressed as $f(x) = 8 - e^{(0.5\ln 2)x}$.

(1 mark)

(ii) Hence, show that $f'(x) = -0.5\ln 2 \times 2^{0.5x}$.

(1 mark)

(c) Using an algebraic process, find the *exact* equation of the tangent to the graph of $y = f(x)$ at P.

(3 marks)