## Question 16 (12 marks)



Consider the function  $f(x) = 4xe^{-x}$  where  $x \ge 0$ . The graph of y = f(x) is shown below.

## (a) (i) Determine the coordinates of the stationary point of the graph of y = f(x). Give your answer correct to three decimal places.

(1 mark)

## (ii) Determine the coordinates of the inflection point of the graph of y = f(x). Give your answer correct to three decimal places.

(2 marks)

The graph of y = f(x) is shown below, along with the tangent to this graph at the point where x = 3.



(b) On the graph above, draw a tangent that has a greater *y*-intercept than that of the tangent shown. (1 mark)

(c) Show that the tangent to the graph of y = f(x) at the point where x = a has the equation



(5 marks)

 $y = 4(1-a)e^{-a}x + 4a^2e^{-a}.$ 

(d) Using the equation given in part (c), determine the value of *a* that maximises the *y*-intercept of the tangent to the graph of y = f(x) at the point where x = a.

(3 marks)