## Question 16 (12 marks)

Consider the function $f(x)=4 x e^{-x}$ where $x \geq 0$. The graph of $y=f(x)$ is shown below.

(a) (i) Determine the coordinates of the stationary point of the graph of $y=f(x)$. Give your answer correct to three decimal places.

(1 mark)
(ii) Determine the coordinates of the inflection point of the graph of $y=f(x)$. Give your answer correct to three decimal places.

(2 marks)

The graph of $y=f(x)$ is shown below, along with the tangent to this graph at the point where $x=3$.

(b) On the graph above, draw a tangent that has a greater $y$-intercept than that of the tangent shown.
(c) Show that the tangent to the graph of $y=f(x)$ at the point where $x=a$ has the equation

$$
y=4(1-a) e^{-a} x+4 a^{2} e^{-a}
$$

(d) Using the equation given in part (c), determine the value of $a$ that maximises the $y$-intercept of the tangent to the graph of $y=f(x)$ at the point where $x=a$.

(3 marks)

