QUESTION 9 (8 marks)

(a) (i) Use integration by parts to find $\int xe^{2x}\mathrm{d}x$.

(3 marks)

(ii) Use integration by parts to show that

$$\int x^2e^{2x}\mathrm{d}x = \frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + c, \text{ where } c \text{ is a constant.}$$

(2 marks)

(b) Let $f(x) = xe^x$.

The graph of $y = f(x)$ for $x \geq 0$ is shown in Figure 8.

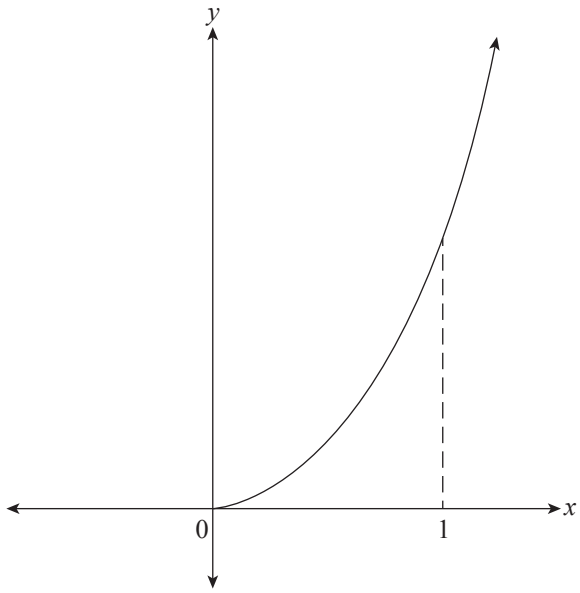

Figure 8

Find the exact volume of the solid obtained when the region bounded by the graph of $f(x)$ on the interval $[0, 1]$ is rotated about the x-axis.

(3 marks)