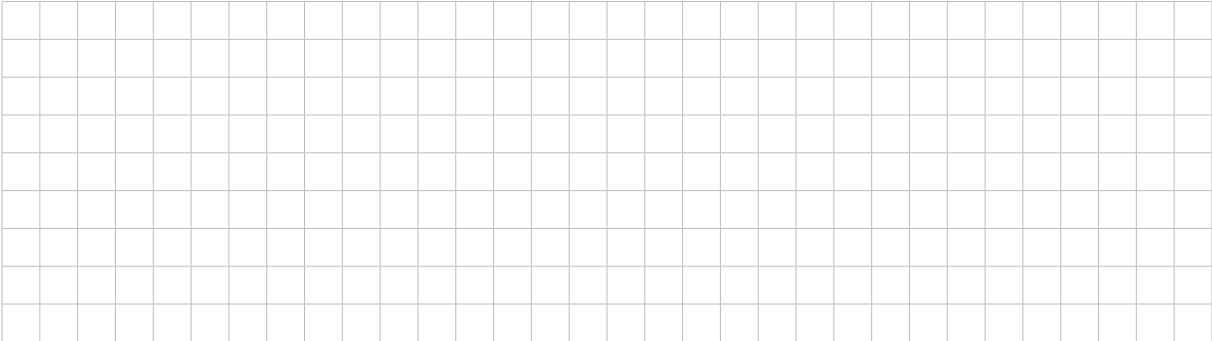


Question 7 (7 marks)

(a) Verify that $\frac{x^2}{x+1} = (x-1) + \frac{1}{x+1}$.

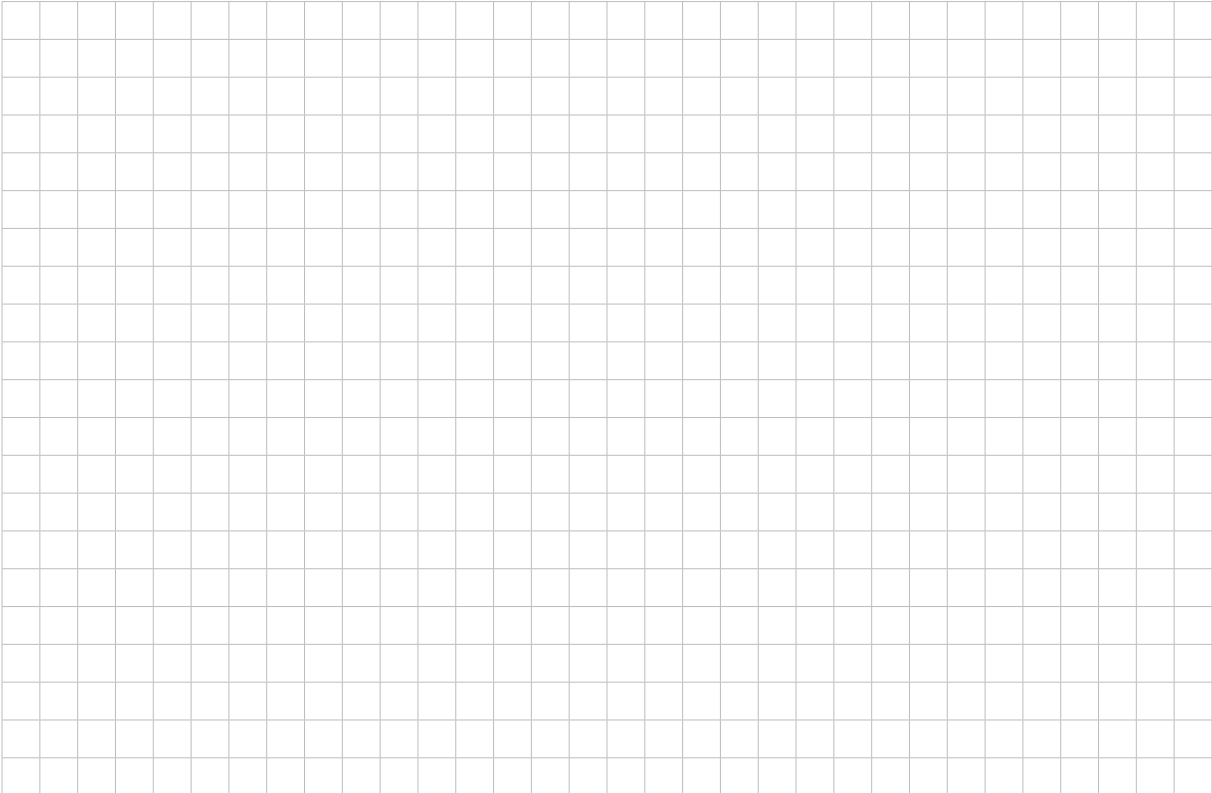


(1 mark)

(b) Use integration by parts to show that, for $x > -1$:

$$\int x \ln(x+1) dx = \frac{x^2}{2} \ln(x+1) - \frac{1}{4}(x-1)^2 - \frac{1}{2} \ln(x+1) + c$$

where c is a constant.



(3 marks)

(c) (i) Figure 8 shows the graph of $f(x) = x \ln(x+1)$ for $x > -1$.

On the same axes, sketch the graph of $g(x) = x |\ln(x+1)|$ for $x > -1$.

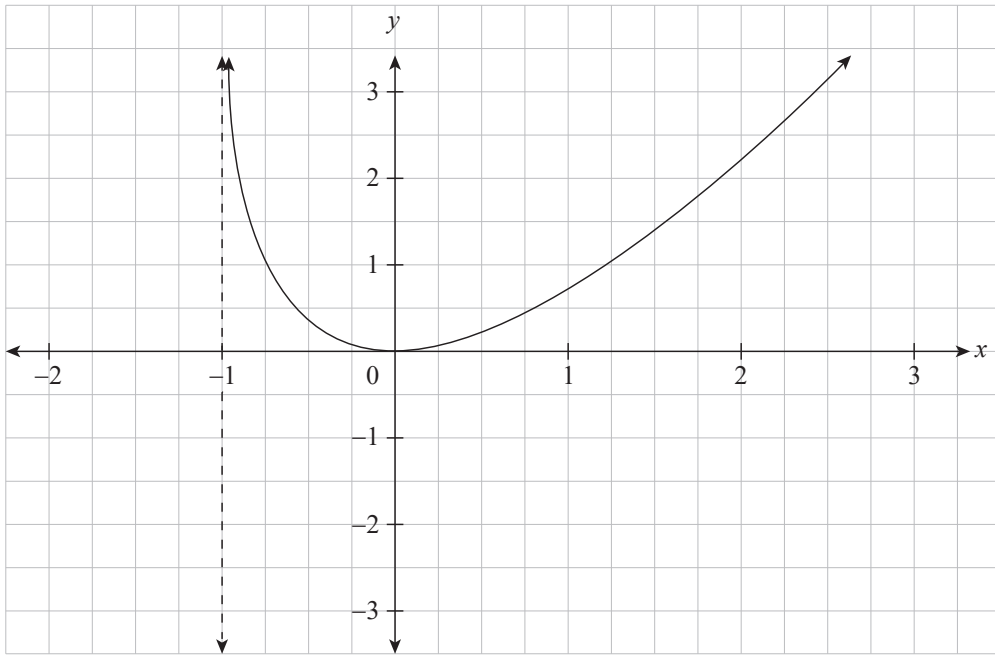


Figure 8

(1 mark)

(ii) Using the information in part (b), find the exact value of $\int_{-\frac{1}{2}}^0 g(x) dx$.



(2 marks)