Question 6 (6 marks)

(a) Use integration by parts to show that

$$\int \arctan x \, dx = x \arctan x - \frac{1}{2}\ln\left(x^2 + 1\right) + c, \text{ where } c \text{ is a constant.}$$

(2 marks)

(b) Consider the graph of $f(x) = \sqrt{\arctan x}$ for $x \geq 0$, shown in Figure 4.

Figure 4

Consider rotating the graph of $f(x)$ about the x-axis between $x = 0$ and $x = 1$.

(i) Show that the volume of the solid that is obtained by this rotation is given by the equation below.
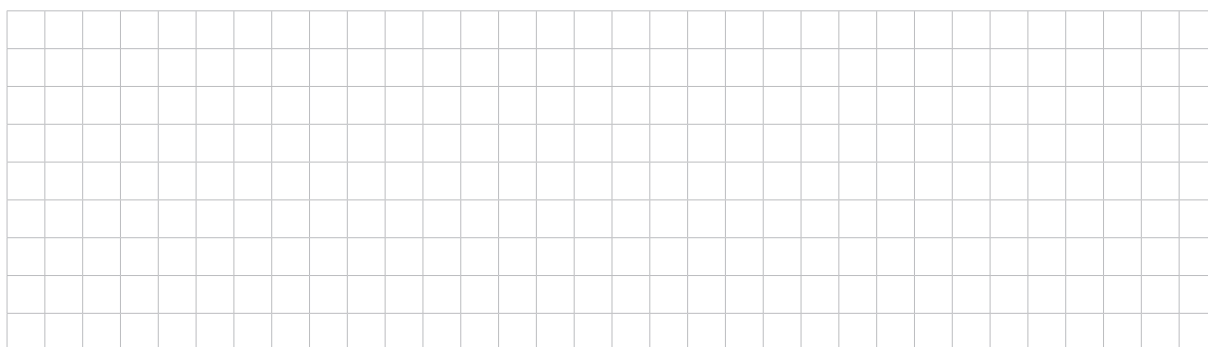
$$V = \pi \int_0^1 \arctan x \, dx$$

(1 mark)

(ii) Hence find the **exact** volume of this solid.

(3 marks)