## Question 4 (10 marks)

Consider the function $f(x)=\frac{x^{3}-2 x+5}{x^{2}+1}$.
(a) Use a division process to show that $f(x)=x-\frac{3 x-5}{x^{2}+1}$.

(b) On the axes in Figure 3, draw the function $f(x)=x-\frac{3 x-5}{x^{2}+1}$.

Clearly show the behaviour of the function near any asymptotes.


Figure 3
(c) Find $g(f(x))$, given $g(x)=\sqrt{x}$ and $f(x)=x-\frac{3 x-5}{x^{2}+1}$.

(1 mark)

The graph of $y=g(f(x))$ is shown in Figure 4.


Figure 4
(d) Consider the solid obtained by rotating the graph of $y=g(f(x))$ about the $x$-axis between $x=-1$ and $x=1$.
(i) Show that the volume of this solid is given by the equation

$$
V=\pi \int_{-1}^{1}\left(x-\frac{3 x}{x^{2}+1}+\frac{5}{x^{2}+1}\right) \mathrm{d} x
$$


(1 mark)
(ii) Show that the exact volume of this solid is $\frac{5 \pi^{2}}{2}$.

(3 marks)

