Question 1 (6 marks)

(a) Use integration by parts to find $\int x \sin x \, dx$.

Let $u = x$ and $v' = \sin x$

Then $u' = 1$ and $v = -\cos x$

$\therefore \int x \sin x \, dx = -x\cos x + \int \cos x \, dx$

$= -x\cos x + \sin x + c$

(3 marks)

Consider the image of a pearl shown in Figure 1.

Figure 2 shows the graph of $y = \sqrt{x \sin x}$, for $0 \le x \le \pi$, which models the top half of the cross-section of this pearl, outlined in Figure 1.

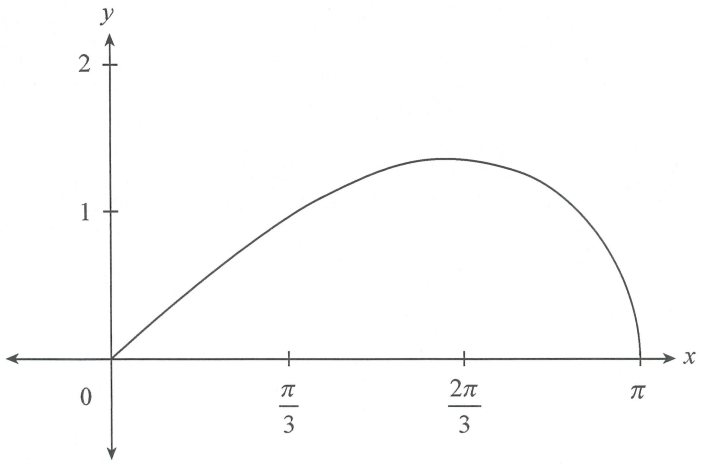

Figure 2

(b) The shape of the pearl can be obtained by rotating the curve in Figure 2 about the x-axis for $0 \leq x \leq \pi$.

Show that, according to the model, the exact volume of the pearl is π^2 cubic units.

$$V = \pi \int_0^{\pi} x \sin x \, dx$$

$$= \pi \left[-x \cdot \cos x + \sin x \right]_0^{\pi}$$

$$= \pi \left(-\pi \cdot -1 + 0 \right) - \pi \left(0 \cdot 1 + 0 \right)$$

$$= \pi^2 \text{ units}^3$$

(3 marks)