

Question 4 (10 marks)

Consider the function $f(x) = \frac{x^3 - 2x + 5}{x^2 + 1}$.

(a) Use a division process to show that $f(x) = x - \frac{3x - 5}{x^2 + 1}$.



(2 marks)

(b) On the axes in Figure 3, draw the function $f(x) = x - \frac{3x - 5}{x^2 + 1}$.

Clearly show the behaviour of the function near any asymptotes.

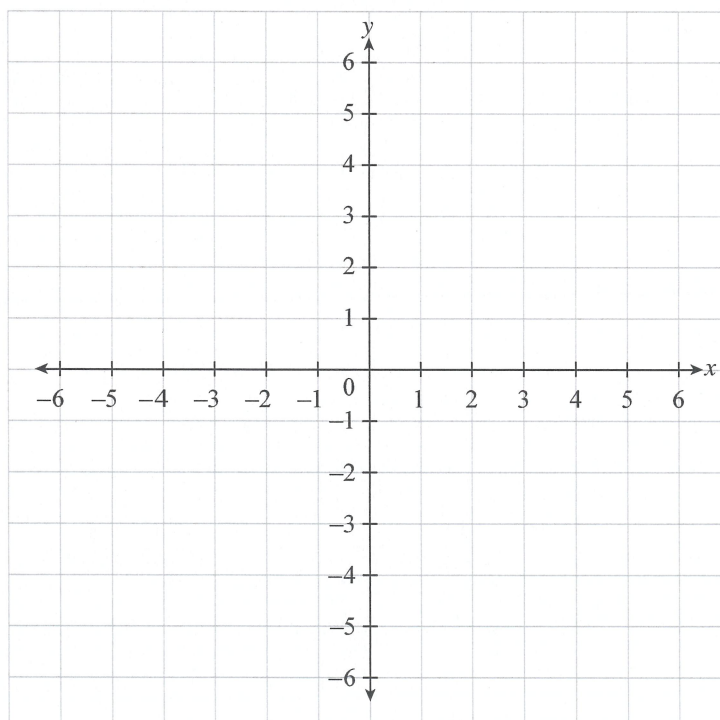


Figure 3

(3 marks)

(ii) Show that the **exact** volume of this solid is $\frac{5\pi^2}{2}$.

$$\begin{aligned} V &= \pi \left[\frac{x^2}{2} - \frac{3}{2} \ln(x^2+1) + 5 \arctan x \right]_1^{-1} \\ &= \pi \left(\frac{1}{2} - \frac{3}{2} \ln 2 + \frac{5\pi}{4} \right) - \pi \left(\frac{1}{2} - \frac{3}{2} \ln 2 + \frac{5\pi}{4} \right) \\ &= \frac{5\pi^2}{2} \text{ units}^3 \end{aligned}$$

(3 marks)