Question 6 (10 marks)

(a) Use integration by parts to show that $\int \arccos x \, dx = x \arccos x - \sqrt{1-x^2} + c$.

Let $u = \arccos x$ and $v' = 1$

Then $u' = \frac{-1}{\sqrt{1-x^2}}$ and $v = x$

$\therefore \int \arccos x \, dx = x \cdot \arccos x - \frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}} \, dx$ Let $u = 1-x^2$ then $\frac{du}{dx} = -2x$

$= x \cdot \arccos x - \frac{1}{2}\int u^{-1/2} \cdot \frac{du}{dx} \cdot dx = x \cdot \arccos x - u^{1/2} + c$

$= x \cdot \arccos x - \sqrt{1-x^2} + c$

(2 marks)

(b) (i) On the axes in Figure 5, draw and label the graph of $f(x) = \arccos x - \frac{\pi}{2}$ for $-1 \le x \le 1$.

(2 marks)

(ii) On the axes in Figure 5, draw and label the graph of $y = |f(x)|$ for $-1 \le x \le 1$. (1 mark)

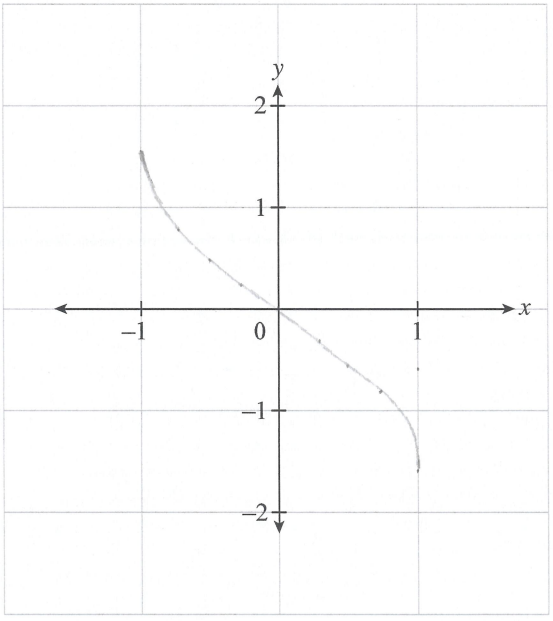

Figure 5

(iii) On the axes in Figure 6, draw the graph of $y = f(|x|)$ for $-1 \leq x \leq 1$.

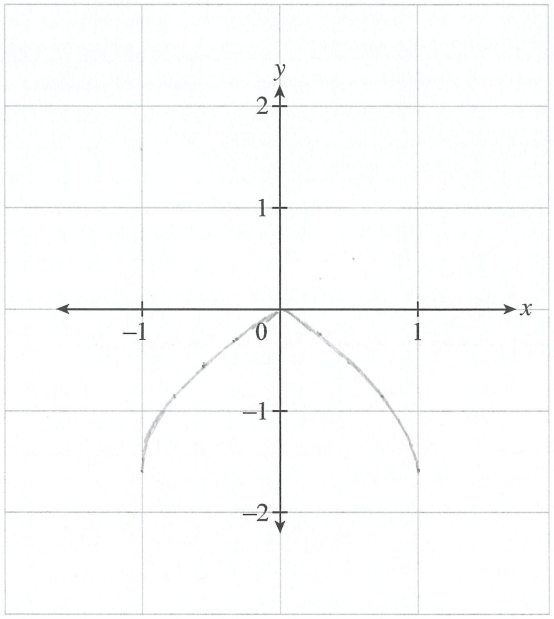

Figure 6

(1 mark)

(c) Using part (a) and part (b)(iii), show that the area between the graph of $y = f(|x|)$ and the y-axis for $0 \leq x \leq 1$ is 1 square unit.

$$A = \int_0^1 \left(\arccos x - \frac{\pi}{2}\right) - \left(-\frac{\pi}{2}\right) dx$$

$$= \left[x.\arccos x - \sqrt{1-x^2} \right]_0^1$$

$$= \left(1.0 - \sqrt{0}\right) - \left(0.1 - \sqrt{1}\right)$$

$$= 1 \text{ units}^2$$

(4 marks)