Question 9 (15 marks)

Consider $f(x)=\dfrac{x^2-1}{x+2}$.

(a) Show that $f(x)=x-2+\dfrac{3}{x+2}$.

$$\frac{x^2-1}{x+2} = \frac{x^2-4+3}{x+2}$$

$$= \frac{\cancel{(x+2)}(x-2)}{\cancel{x+2}} + \frac{3}{x+2} = x-2+\frac{3}{x+2}$$

(1 mark)

(b) Sketch the graph of $y=f(x)$ on Figure 7 below.

Clearly label all asymptotes and the axes intercepts.

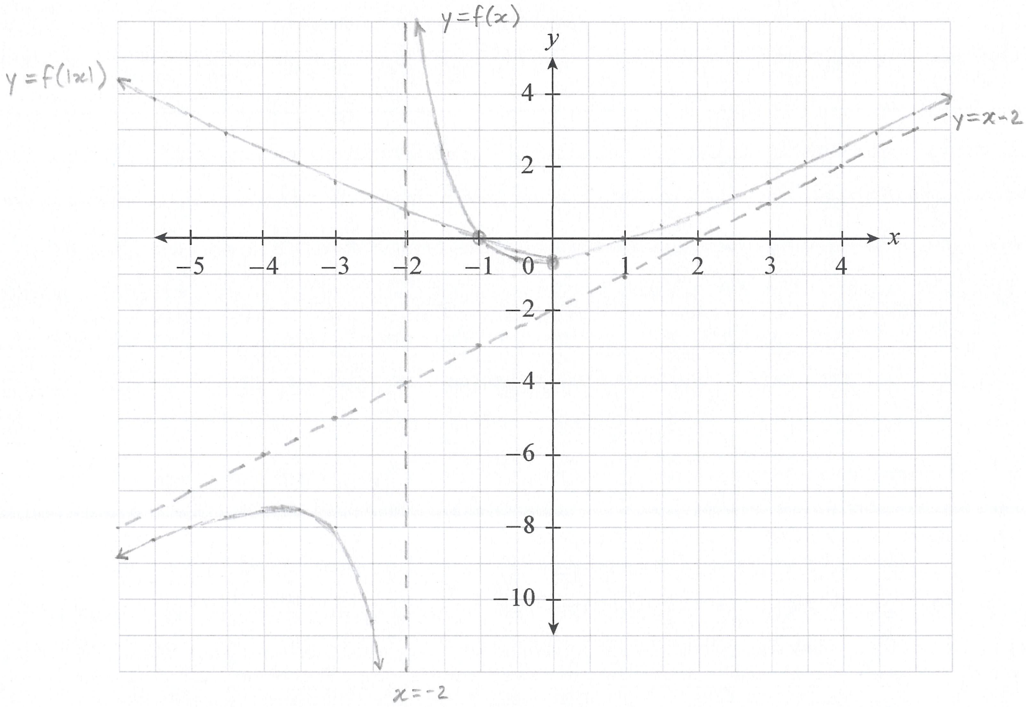

Figure 7 (4 marks)

(c) (i) On Figure 7 above, sketch and clearly label the graph of $y=f(|x|)$. (2 marks)

(ii) State the interval for which $f(|x|)>f(x)$ for $x>-2$.

$-1<x<0$

(1 mark)

(d) (i) Show that the expression for finding the area between $f(|x|)$ and $f(x)$ for $x > -2$ is given by

$$\int_{-1}^{0} -2x + \frac{6x}{4-x^2}\,dx.$$

Note that $|x| = -x$ for $x \le 0$.

$$A = \int_{-1}^{0} f(|x|) - f(x)\,dx$$

$$= \int_{-1}^{0} \left(-x - 2 + \frac{3}{-x+2}\right) - \left(x - 2 + \frac{3}{x+2}\right) dx \qquad \{\text{using parts (a) and (c) above}\}$$

$$= \int_{-1}^{0} -2x + \frac{3(2+x) - 3(2-x)}{(2-x)(2+x)}\,dx$$

$$= \int_{-1}^{0} -2x + \frac{6x}{4-x^2}\,dx$$

(4 marks)

(ii) Hence show that the exact value of the area between $f(|x|)$ and $f(x)$ is

$$1 + 3\ln\left(\frac{3}{4}\right).$$

$$A = \int_{-1}^{0} -2x - 3 \cdot \frac{-2x}{4-x^2}\,dx$$

$$= \left[-x^2 - 3\ln|4-x^2|\right]_{-1}^{0}$$

$$= (0 - 3\ln 4) - (-1 - 3\ln 3)$$

$$= 1 - 3\ln\left(\frac{3}{4}\right) \text{ units}^2$$

(3 marks)