PART 1 (Questions 1 to 10)

(75 marks)

Question 1 (5 marks)

(a) Show that $\frac{4}{x^2-4}=\frac{1}{x-2}-\frac{1}{x+2}$.

$\frac{1}{x-2} - \frac{1}{x+2} = \frac{(x+2)-(x-2)}{(x-2)(x+2)}$

$= \frac{4}{x^2-4}$

(1 mark)

(b) (i) Hence show that $\int \frac{1}{x^2-4}\,dx=\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+c$.

$\therefore \int \frac{1}{x^2-4}\,dx = \frac{1}{4}\int \frac{1}{x-2} - \frac{1}{x+2}\,dx$ {using (a) above}

$= \frac{1}{4}\ln|x-2| - \frac{1}{4}\ln|x+2| + c$

$= \frac{1}{4}\ln\frac{|x-2|}{|x+2|} + c$

$= \frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| + c$

(3 marks)

(ii) Find the exact value of $\int_0^1 \frac{1}{x^2-4}\,dx$.

$$\int_0^1 \frac{1}{x^2-4}\,dx = \left[\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|\right]_0^1$$

$$= \left(\frac{1}{4}\ln\frac{1}{3}\right) - \left(\frac{1}{4}\ln 1\right)$$

$$= -\frac{1}{4}\ln 3$$

(1 mark)