QUESTION 8 (10 marks)

(a) Show that $\frac{1}{x-2}-\frac{1}{x+3}=\frac{5}{(x-2)(x+3)}$.

(1 mark)

Let $f(x)=\frac{5}{(x-2)(x+3)}$.

(b) (i) Draw the graph of $y=f(x)$ on the axes in Figure 5.

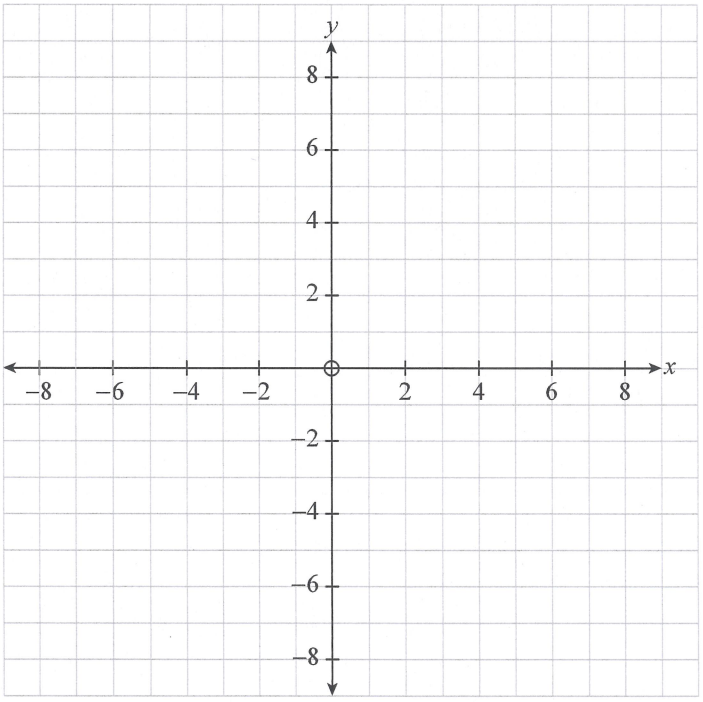

Figure 5

(3 marks)

(ii) Draw the graph of $y = \left| f(x) \right|$ on the axes in Figure 6.

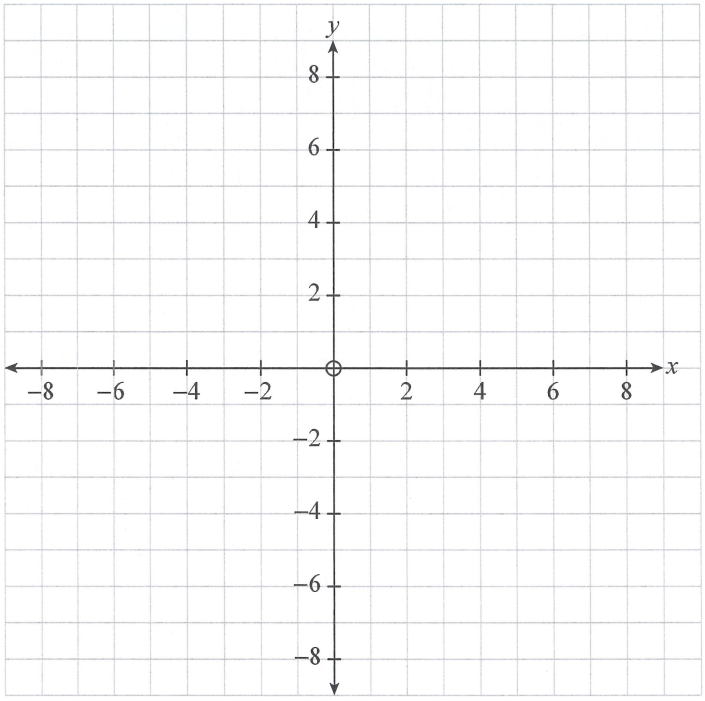

Figure 6

(1 mark)

(iii) Draw the graph of $y = \left| f(x) \right| - f(x)$ on the axes in Figure 7.

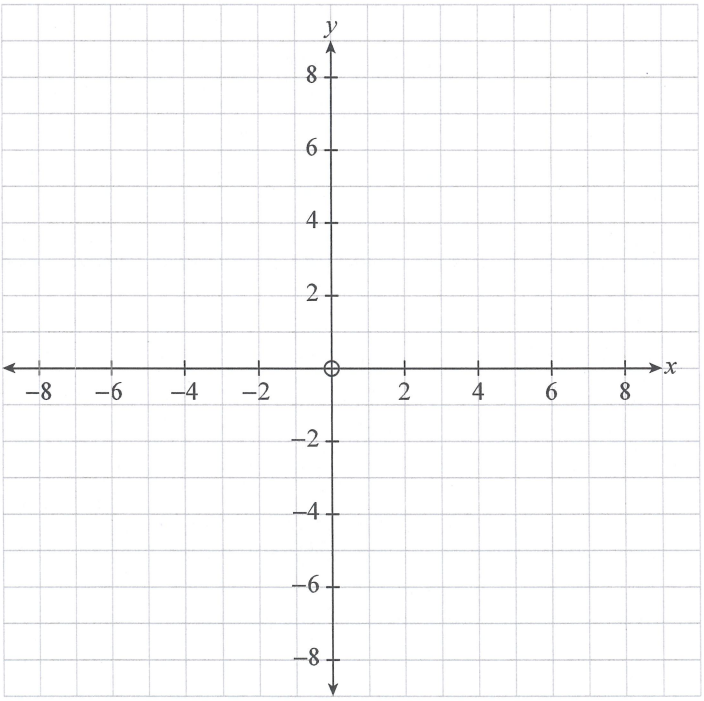

Figure 7

(2 marks)

(c) Find the exact area between the graph of $y = |f(x)| - f(x)$, the x-axis, and the lines $x = -2$ and $x = 1$.

$$\text{Area} = \int_{-2}^{1} \frac{-10}{(x-2)(x+3)}\,dx$$

$$= -2\int_{-2}^{1} \frac{1}{x-2} - \frac{1}{x+3}\,dx \qquad \{\text{using part (a) above}\}$$

$$= -2\Big[\ln|x-2| - \ln|x+3|\Big]_{-2}^{1}$$

$$= -2\Big[(\ln 1^{0} - \ln 4) - (\ln 4 - \ln 1^{0})\Big]$$

$$= 4\ln 4 \text{ units}^2$$

(3 marks)