QUESTION 9 (8 marks)

(a) (i) Use integration by parts to find $\int xe^{2x}\mathrm{d}x$.

Let $u = x$ and $v' = e^{2x}$

Then $u' = 1$ and $v = \frac{1}{2}e^{2x}$

$\therefore \int x.e^{2x}\,dx = \frac{x}{2}.e^{2x} - \int \frac{1}{2}.e^{2x}\,dx$

$= \frac{x}{2}.e^{2x} - \frac{e^{2x}}{4} + c$

(3 marks)

(ii) Use integration by parts to show that

$$\int x^2e^{2x}\mathrm{d}x = \frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + c\text{, where } c \text{ is a constant.}$$

Let $u = x^2$ and $v' = e^{2x}$

Then $u' = 2x$ and $v = \frac{1}{2}e^{2x}$

$\therefore \int x^2 e^{2x}\,dx = \frac{x^2}{2}.e^{2x} - \int xe^{2x}\,dx$

$= \frac{x^2}{2}.e^{2x} - \frac{x}{2}.e^{2x} + \frac{e^{2x}}{4} + c$ {using the result from (a)(i) above}

(2 marks)

(b) Let $f(x) = xe^x$.

The graph of $y = f(x)$ for $x \geq 0$ is shown in Figure 8.

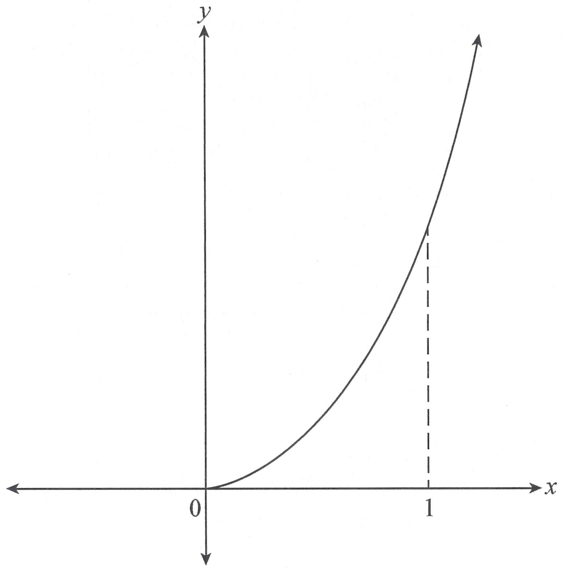

Figure 8

Find the exact volume of the solid obtained when the region bounded by the graph of $f(x)$ on the interval $[0, 1]$ is rotated about the x-axis.

$$V = \pi \int_0^1 x^2 . e^{2x} \, dx$$

$$= \pi \left[\frac{x^2}{2} . e^{2x} - \frac{x}{2} . e^{2x} + \frac{e^{2x}}{4} \right]_0^1$$

$$= \pi \left(\frac{e^2}{4} - \frac{1}{4} \right)$$

$$= \frac{\pi}{4} \left(e^2 - 1 \right) \text{ units}^3$$

(3 marks)

 PLEASE TURN OVER