**Stage 2 Specialist Mathematics**

**Integration Techniques and Applications Test**

**Topic 5: Subtopics 5.1, 5.2**

**Total Marks - 28**

**(Calculator and one A4 page of handwritten notes permitted.)**

1. (8 marks)

Select the relevant integration rule or technique to integrate the following:

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(2 marks)

1. (6 marks)
2. Find the values of and such that

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(2 marks)

1. Hence or otherwise, find

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(4 marks)

1. (8 marks)
2. Use integration by parts to find .

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(3 marks)

* 1. Use integration by parts to show that , where is a constant.

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(2 marks)

1. Let .

The graph of for is shown below.

Find the exact volume of the solid obtained when the region bounded by the graph of on the interval is rotated about the x-axis.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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(3 marks)

1. (6 marks)

Let .

1. Find .

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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(2 marks)

The following diagram shows part of the graph of .

1. The shaded region is enclosed by the graph of , the x-axis, and the lines and . This region has an area of . Find the value of .

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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(4 marks)