**Stage 2 Specialist Mathematics**

**Assessment Type 2: Mathematical Investigation**

**Topic 4 – Vectors in Three Dimensions**

**The Escaped Robotic Bees**

Buzz labs have built a prototype of robotic bees. They are still in early stages of development and they will immediately sting any person they come in to contact with.

Two bees are running loose in the labs and are heading straight towards the engineers. They have two ways to stop the bees before being stung: hitting them with a laser beam or hitting them with a disabling sonic blast.

In this investigation you will use vectors to represent the position and velocity of the two robotic bees in 3-dimensional space.

Considering two bees, A and B, set up the following:

* Bee A has initial position $\left(a\_{1},a\_{2},a\_{3}\right)$ and has velocity vector $\left[v\_{1},v\_{2},v\_{3}\right]$
* Bee B has initial position $\left(b\_{1},b\_{2},b\_{3}\right)$ and has velocity vector $\left[w\_{1},w\_{2},w\_{3}\right]$

In this investigation, using varying initial positions and velocity vectors for your bees, you investigate the location and speed of the bees and the distance they are apart.

In addition, you need to determine the best position and time to use the laser beam and sonic blaster to disable the bees. You will model your scenarios with 3D graphing software.

**PART 1**

Sensibly choose the initial positions and velocity vectors of your bees. You must ensure that:

* The speed of each bee is within a realistic range
* The motion of the bees are not parallel or coincident to each other
* The shortest distance between the two bees does not occur when $t=0$

You will create a minimum of 2 scenarios for the velocities of the bees: one where they are coplanar and one where they are skew.

For your chosen values find the parametric equations for the position of each bee at time $t$, where $t$ is the time in seconds, using *metres* for the distance.

What is the speed of each bee and how long does it take each bee to reach an engineer at a point of your choice along its straight-line path?

**PART 2**

If at time *t*, bee A is at position P and bee B is at position Q, determine the vector $\vec{PQ}$ and find their distance apart at any instant. Find the time when the bees are closest to each other and their distance apart at this instant.

**PART 3**

Laser Beam:

Now investigate the best location and time for the laser beam to hit both bees with one shot. The laser beam may be represented by a ray in 3D, or a vector.

Sonic Blaster:

Also investigate the best location and time for a sonic blaster to hit both bees in one shot. Add the blast to your 3D model.

Now come up with your own method(s) on how to disable and stop the two bees and include it in your 3D model. Be as creative as you like.

**PART 4**

Analyse and interpret your results, including consideration of the reasonableness and limitations of the results. Describe and discuss any assumptions you have made. Using GeoGebra, or some other 3D modelling software, develop a graphical representation of the motion of the bees, laser beams, sonic blaster, engineers and any other features you have included.

**Complete a report for the mathematical investigation.**

**The investigation report should be a maximum of 15 single-sided A4 pages if written, or the equivalent in multimodal form.**

The report may take a variety of forms, but would usually include the following:

* an outline of the problem and context
* the method required to find a solution, in terms of the mathematical model or strategy used
* the application of the mathematical model or strategy, including
* relevant data and/or information
* mathematical calculations and results, using appropriate representations
* the analysis and interpretation of results, including consideration of the reasonableness and limitations of the results
* the results and conclusions in the context of the problem.

A bibliography and appendices, as appropriate, may be used.

The format of an investigation report may be written or multimodal.

The report, excluding bibliography and appendices if used, must be a maximum of 15 A4 pages if written, or the equivalent in multimodal form. The maximum page limit is for single-sided A4 pages with minimum font size 10. Page reduction, such as 2 A4 pages reduced to fit on 1 A4 page, is not acceptable. Conclusions, interpretations and/or arguments that are required for the assessment must be presented in the report, and not in an appendix. Appendices are used only to support the report, and do not form part of the assessment decision.

**Assessment Design Criteria**

**Concepts and Techniques**

The specific features are as follows:

CT1 Knowledge and understanding of concepts and relationships.

CT2 Selection and application of mathematical techniques and algorithms to find solutions to problems in a variety of contexts.

CT3 Application of mathematical models.

**Reasoning and Communication**

The specific features are as follows:

RC1 Interpretation of mathematical results.

RC2 Drawing conclusions from mathematical results, with an understanding of their reasonableness and limitations.

RC3 Use of appropriate mathematical notation, representations, and terminology.

RC4 Communication of mathematical ideas and reasoning to develop logical arguments.

Performance Standards for Stage 2 Specialist Mathematics

| - | Concepts and Techniques | Reasoning and Communication |
| --- | --- | --- |
| A | Comprehensive knowledge and understanding of concepts and relationships.Highly effective selection and application of mathematical techniques and algorithms to find efficient and accurate solutions to routine and complex problems in a variety of contexts.Successful development and application of mathematical models to find concise and accurate solutions.Appropriate and effective use of electronic technology to find accurate solutions to routine and complex problems. | Comprehensive interpretation of mathematical results in the context of the problem.Drawing logical conclusions from mathematical results, with a comprehensive understanding of their reasonableness and limitations.Proficient and accurate use of appropriate mathematical notation, representations, and terminology.Highly effective communication of mathematical ideas and reasoning to develop logical and concise arguments.Effective development and testing of valid conjectures, with proof. |
| B | Some depth of knowledge and understanding of concepts and relationships.Mostly effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine and some complex problems in a variety of contexts.Some development and successful application of mathematical models to find mostly accurate solutions.Mostly appropriate and effective use of electronic technology to find mostly accurate solutions to routine and some complex problems. | Mostly appropriate interpretation of mathematical results in the context of the problem.Drawing mostly logical conclusions from mathematical results, with some depth of understanding of their reasonableness and limitations.Mostly accurate use of appropriate mathematical notation, representations, and terminology.Mostly effective communication of mathematical ideas and reasoning to develop mostly logical arguments.Mostly effective development and testing of valid conjectures, with substantial attempt at proof. |
| C | Generally competent knowledge and understanding of concepts and relationships.Generally effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine problems in a variety of contexts.Successful application of mathematical models to find generally accurate solutions.Generally appropriate and effective use of electronic technology to find mostly accurate solutions to routine problems. | Generally appropriate interpretation of mathematical results in the context of the problem.Drawing some logical conclusions from mathematical results, with some understanding of their reasonableness and limitations.Generally appropriate use of mathematical notation, representations, and terminology, with reasonable accuracy.Generally effective communication of mathematical ideas and reasoning to develop some logical arguments.Development and testing of generally valid conjectures, with some attempt at proof. |
| D | Basic knowledge and some understanding of concepts and relationships.Some selection and application of mathematical techniques and algorithms to find some accurate solutions to routine problems in some contexts.Some application of mathematical models to find some accurate or partially accurate solutions.Some appropriate use of electronic technology to find some accurate solutions to routine problems. | Some interpretation of mathematical results.Drawing some conclusions from mathematical results, with some awareness of their reasonableness or limitations.Some appropriate use of mathematical notation, representations, and terminology, with some accuracy.Some communication of mathematical ideas, with attempted reasoning and/or arguments.Attempted development or testing of a reasonable conjecture. |
| E | Limited knowledge or understanding of concepts and relationships.Attempted selection and limited application of mathematical techniques or algorithms, with limited accuracy in solving routine problems.Attempted application of mathematical models, with limited accuracy.Attempted use of electronic technology, with limited accuracy in solving routine problems. | Limited interpretation of mathematical results.Limited understanding of the meaning of mathematical results, their reasonableness, or limitations.Limited use of appropriate mathematical notation, representations, or terminology, with limited accuracy.Attempted communication of mathematical ideas, with limited reasoning.Limited attempt to develop or test a conjecture. |