Question 8 (10 marks)

Consider the continuous random variable X, with the probability density function $f(x) = \frac{1}{8}x$ for $0 \leq x \leq 4$. A graph of $y = f(x)$ is shown in Figure 9.

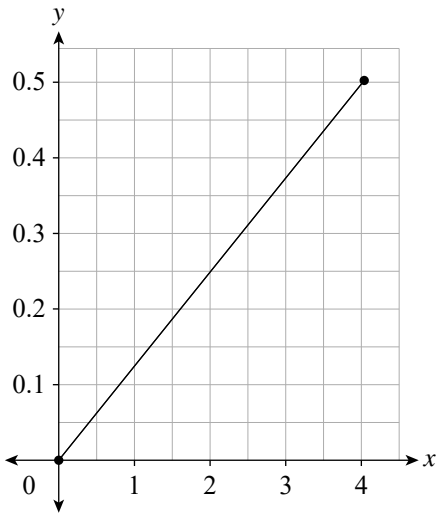

Figure 9

(a) Given $\mu_X = \frac{8}{3}$, calculate σ_X.

(2 marks)

(b) Find $\Pr(2 \leq X \leq 3)$.

(1 mark)

(c) Using integration and an algebraic process, show that $\Pr(0 \le X \le 1) = \frac{1}{16}$.

(2 marks)

Consider the real numbers m and n, such that $\Pr(m \le X \le n) = \frac{1}{16}$ where $0 \le m \le 4$ and $0 \le n \le 4$.

The following conjecture is made for the value of n in terms of m:

$$n = \sqrt{m^2 + 1}.$$

(d) Prove this conjecture.

(3 marks)

(e) Use the conjecture $n = \sqrt{m^2 + 1}$ to determine the *exact* maximum value of m that satisfies the probability statement $\Pr(m \leq X \leq n) = \frac{1}{16}$, for $0 \leq x \leq 4$.

(2 marks)