Question 4 (9 marks)

The fat content in grams, X, per carton of Ted's Oat Milk can be modelled using a normal distribution with a mean of $\mu_X = 2.7\,\text{g}$ and a standard deviation of $\sigma_X = 0.35\,\text{g}$.

(a) Determine the probability that a carton of Ted's Oat Milk will contain:

(i) at least $2.8\,\text{g}$ of fat

(1 mark)

(ii) between $1\,\text{g}$ and $3\,\text{g}$ of fat.

(1 mark)

(b) Ted's Oat Milk is sold in packs of six cartons. Let S_6 represent the distribution of fat content in one six-pack of Ted's Oat Milk.

(i) Explain why S_6 can be modelled by a normal distribution.

(1 mark)

(ii) Show that the distribution of S_6 has a mean of $\mu_{S_6} = 16.2\,\text{g}$ and a standard deviation of $\sigma_{S_6} = 0.857\,\text{g}$ (correct to three significant figures).

(2 marks)

(iii) Hence, determine the probability that a six-pack of Ted's Oat Milk will contain less than 18 g of fat.

(1 mark)

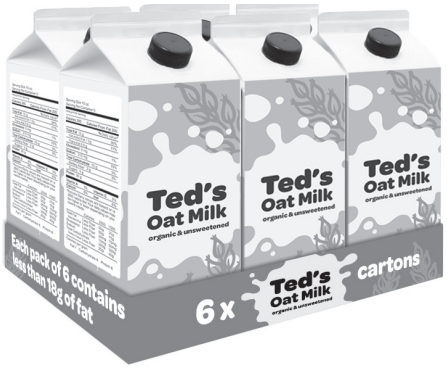
(c) Find the value of k if $\Pr(Z < k) = 0.99$, given that $Z \sim N(0,1)$.

(1 mark)

(d) The label on the side of a six-pack of Ted's Oat Milk states that:

'Each pack of 6 contains less than 18 g of fat.'

The company would like this statement to be true for at least 99% of Ted's Oat Milk six-packs and it is considering improvements to the manufacturing process. Although it cannot change the mean fat content of a six-pack through these improvements, the standard deviation can be reduced.



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Using your answer to part (c), determine the largest possible value of σ_{S_6} that will result in at least 99% of six-packs containing less than 18 g of fat. Assume that $\mu_{S_6} = 16.2$ g.

(2 marks)