**Stage 2 Specialist Mathematics**

**Rates of Change and Differential Equations Test**

**Topic 6: Subtopics 6.1, 6.2, 6.3, 6.4, 6.5**

**Total Marks - 38**

**(Calculator and one A4 page of handwritten notes permitted.)**

1. (8 marks)

A curve has the following parametric equations:

where .

1. Sketch a graph of the curve on the axes in Figure 1.

Figure 1

(3 marks)

1. Show that .

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(3 marks)

1. What is the slope of the tangent to the curve at ?

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(1 mark)

* 1. Draw the tangent to the curve at on your graph in Figure 1.

(1 mark)

1. (8 marks)

Sociologists can study the spread of a new fashion by modelling the rate at which the fashion spreads. For one such model the rate of spread is given by the differential equation

where is the number of people who follow the fashion and is in weeks. Initially there are 100 people following the fashion.

1. Show that .

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(1 mark)

1. Solve the differential equation given above with the initial condition , to show that

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(5 marks)

1. What is the estimated time at which the fashion is spreading at the greatest rate?

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(2 marks)

1. (7 marks)

A pharmaceutical company markets an antibiotic tablet that has the shape of a cylinder with hemispherical ends, as shown in Figure 2. The surface area of the tablet is 200 square millimetres. The cylindrical section has a length of millimetres and a radius of millimetres.

Figure 2

1. Show that the surface area of the tablet is .

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(1 mark)

* 1. Hence show that .

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(3 marks)

At a particular instant when the tablet is dissolving:

* the radius is 1 millimetre and is decreasing at the rate of 0.05 millimetres per second;
* the surface area is half its original value and is decreasing at the rate of 6 square millimetres per second.

1. Find the rate at which the length is changing at this instant.

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(3 marks)

1. (7 marks)

Figure 3 shows the slope field for the differential equation and the point .

Figure 3

1. On Figure 3 draw the solution curve that passes through P.

(3 marks)

1. By solving the differential equation , with the initial condition and , show that .

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(4 marks)

1. (8 marks)

A Bézier curve has control points , , , and , and for this curve .

1. Find for the Bézier curve.

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(2 marks)

1. Find the coordinates of the highest point on the Bézier curve.

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(4 marks)

1. Find the x-intercept for the Bézier curve correct to three significant figures.

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(2 marks)