Question 9 (15 marks)

On the Argand diagram in Figure 10 is the set of complex numbers z such that $z = \text{cis}\theta$ for $-\pi < \theta \leq \pi$.

The Argand diagram in Figure 11 shows the complex numbers z_1, z_2, z_3, z_4, z_5 which are the zeros of the polynomial $z^5 - 1$.

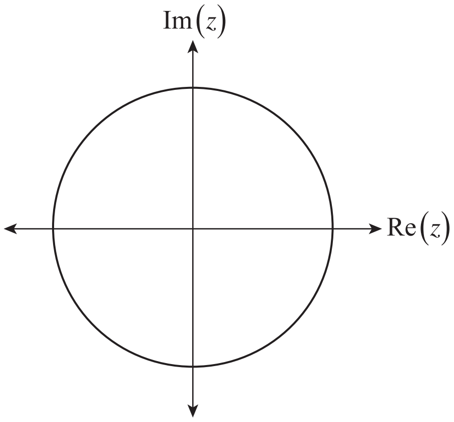

Figure 10

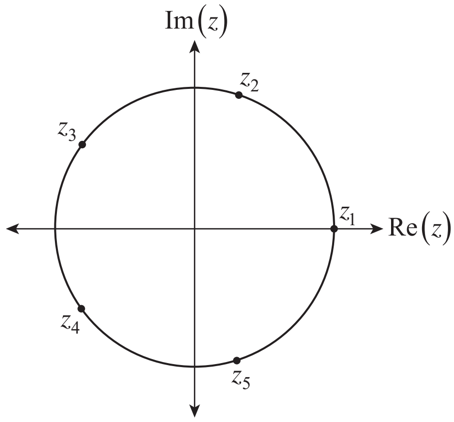

Figure 11

(a) Using De Moivre's theorem or otherwise, write the zeros of $z^5 - 1$ in exact polar form.

(3 marks)

(b) Consider the function $f(z) = z^2 + z + 1 + \frac{1}{z} + \frac{1}{z^2}$, where $z = \text{cis}\theta$.

(i) Show that $|f(z)| \leq 5$.

(2 marks)

(ii) Find a value of z for which $|f(z)| = 5$.

(1 mark)

(c) (i) Show that $z^5 - 1 = z^2(z-1) \times f(z)$.

(2 marks)

(ii) Hence list the four zeros of $f(z)$ in polar form.

(1 mark)

(d) Using $z = \text{cis}\theta$:

(i) show that $f(z)$ is real.

(3 marks)

(ii) show that $f(z) = 4\cos^2\theta + 2\cos\theta - 1$.

(1 mark)

(iii) find the minimum value of $f(z)$.

(1 mark)

(iv) on Figure 12, mark the value(s) of z for which $f(z)$ is the minimum.

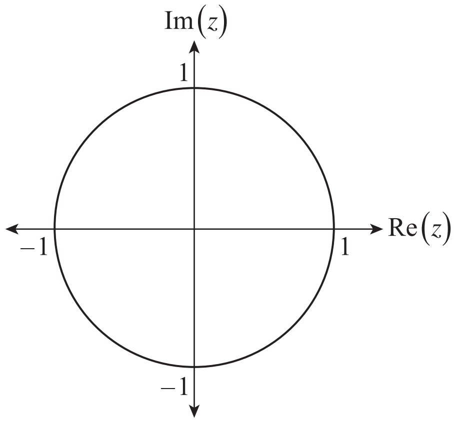

Figure 12

(1 mark)