

Question 3 (7 marks)

(a) Write the following complex numbers in exact polar form.

(i) $z_1 = \sqrt{3} - 3i$

$$z_1 = 2\sqrt{3} \operatorname{cis}\left(-\frac{\pi}{3}\right)$$

(1 mark)

(ii) $z_2 = 3\sqrt{3} - 3i$

$$z_2 = 6 \operatorname{cis}\left(-\frac{\pi}{6}\right)$$

(1 mark)

(b) Complex numbers z_1 and z_2 from part (a) are shown on the Argand diagram in Figure 2. The measure of the acute angle between z_1 and z_2 is θ .

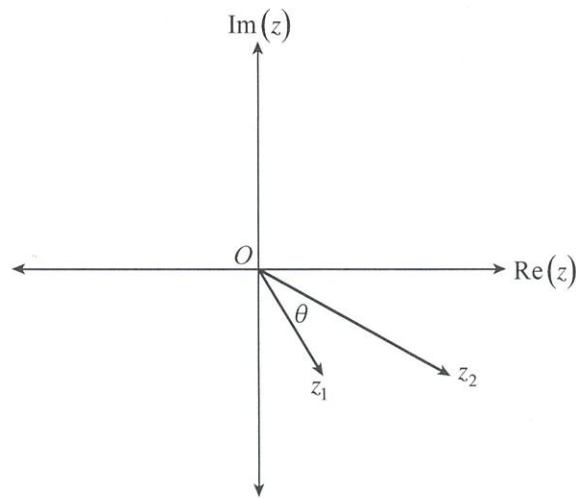


Figure 2

(i) Write $\frac{z_2}{z_1}$ in polar form.

$$\frac{z_2}{z_1} = \sqrt{3} \operatorname{cis}\left(\frac{\pi}{6}\right)$$

(1 mark)

(ii) State the exact value of θ .

$$\theta = \frac{\pi}{6}$$

(1 mark)

(c) The complex number z_2 from part (a) is scaled by a factor of $\frac{1}{2}$ and rotated anticlockwise about the origin O through $\frac{3\theta}{2}$ to produce complex number z_3 , as shown on the Argand diagram in Figure 3.

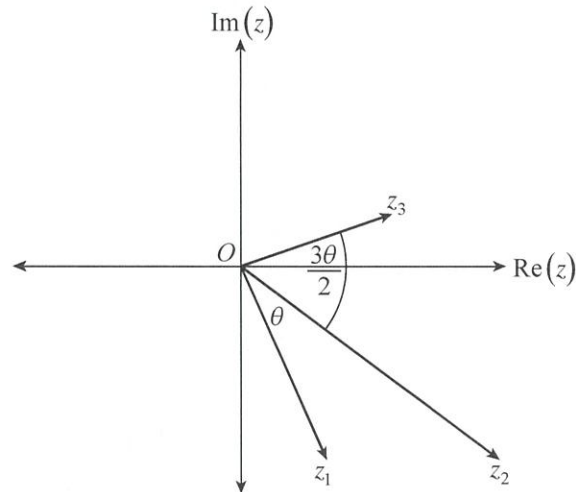


Figure 3

Using the value of θ found in part (b)(ii), find z_3 in exact polar form.

$$\begin{aligned} z_3 &= 6 \operatorname{cis}\left(-\frac{\pi}{6}\right) \cdot \frac{1}{2} \operatorname{cis}\left(\frac{\pi}{4}\right) \\ &= 3 \operatorname{cis}\left(\frac{\pi}{12}\right) \end{aligned}$$

(3 marks)