Question 7 (7 marks)

Consider the complex numbers $z = i$ and $w = \dfrac{\sqrt{2}}{1-i}$.

(a) (i) Express z in exact polar form.

$z = \text{cis}\left(\frac{\pi}{2}\right)$

(1 mark)

(ii) Express w in exact polar form.

$z = \text{cis}\left(\frac{\pi}{4}\right)$

(1 mark)

(b) On the Argand diagram in Figure 6, draw and label z, w, and $z + w$.

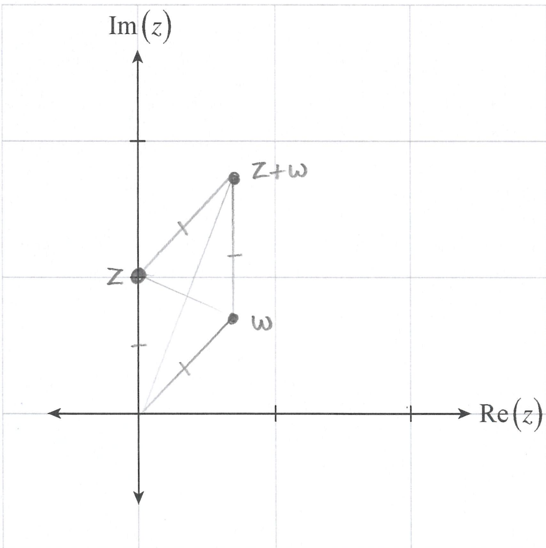

Figure 6

(2 marks)

(c) (i) Using z, w, and $z+w$ from part (b), show that $\arg(z+w)=\frac{3\pi}{8}$.

$\angle WOZ = \frac{\pi}{4}$

Because 0, z, z+w, and w form a rhombus, $\arg(z+w) = \arg(w) + \frac{1}{2}\angle WOZ$

$= \frac{\pi}{4} + \frac{\pi}{8}$

$= \frac{3\pi}{8}$

(1 mark)

(ii) Write $z+w$ in Cartesian form.

$z + w = i + \frac{1}{\sqrt{2}} + i \cdot \frac{1}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}} + i\left(1 + \frac{1}{\sqrt{2}}\right)$

(1 mark)

(iii) Hence show that $\tan\frac{3\pi}{8}=1+\sqrt{2}$.

$\therefore \tan\frac{3\pi}{8} = \frac{1 + 1/\sqrt{2}}{1/\sqrt{2}}$

$= \frac{\sqrt{2}+1}{1}$

$= 1 + \sqrt{2}$

(1 mark)

 PLEASE TURN OVER