South Australian Certificate of Education

General Mathematics
2020

Question booklet

- Questions 1 to 9
- Answer ***all*** questions
- Write your answers in this question booklet
- You may write on page 21 if you need more space

Examination information

Materials
- Question booklet
- SACE registration number label

Instructions
- Show appropriate working and steps of logic in this question booklet
- Use black or blue pen
- You may use a sharp dark pencil for diagrams and graphical representations
- Approved calculators may be used — complete the box below

Total time: 130 minutes
Total marks: 90

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Attach your SACE registration number label here

Graphics calculator

1. Brand _____

Model _____

2. Brand _____

Model _____

Government of South Australia

Question 1 (11 marks)

The speeds of cars travelling on a section of a freeway have been found to be normally distributed with a mean of 95 km/h and a standard deviation of 3 km/h.

(a) Calculate the percentage of cars expected to have a speed between 89 km/h and 98 km/h.

(1 mark)

(b) A traffic analyst estimates that 20% of cars travel too fast on this section of the freeway.

Calculate the slowest speed that would be considered too fast by the analyst.

(2 marks)

The speed limit on this section of the freeway is 100 km/h. A speed camera has been installed.

(c) (i) What percentage of cars would be expected to be exceeding the speed limit?

(1 mark)

(ii) If $15\,340$ cars have passed the camera, how many cars would be expected to have been exceeding the speed limit?

(1 mark)

(d) Tick *one* box below to complete the following statement.

The estimation made by the traffic analyst in part (b) was:

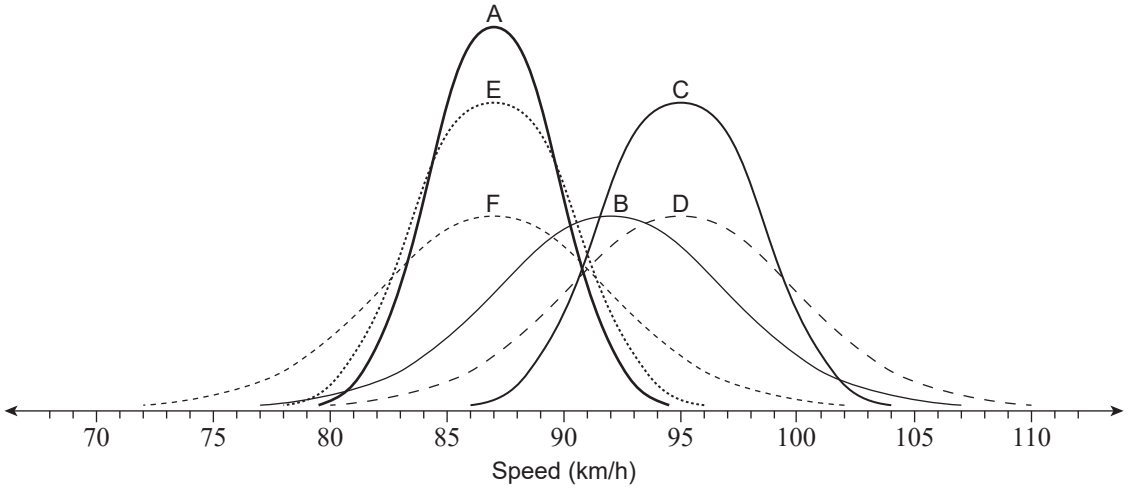
realistic ☐ unrealistic ☐

Justify your selection using answers from part (b) and/or part (c).

(2 marks)

(e) Which curve on the graph below best represents the distribution of speeds of cars on this section of the freeway? Tick the appropriate box to indicate your answer.

A ☐ B ☐ C ☐ D ☐ E ☐ F ☐ (1 mark)



The speeds of heavy goods vehicles (HGVs) travelling on the same section of the freeway have been found to be normally distributed with a mean of 87 km/h and a standard deviation of 5 km/h.

(f) Which curve on the graph above best represents the distribution of the speeds of HGVs on this section of the freeway? Tick the appropriate box to indicate your answer.

A ☐ B ☐ C ☐ D ☐ E ☐ F ☐ (1 mark)

(g) From the curves on the graph above:

(i) identify any *two* distributions with the same mean. _____ (1 mark)

(ii) identify any *two* distributions with the same standard deviation. _____ (1 mark)

Question 2 (12 marks)

Yusuf is 23 years old and currently earns $\$32\,000$ per year. He is setting up his superannuation account which earns 6.93% interest per annum, compounded quarterly. Yusuf's employer will contribute 11% of his salary and Yusuf plans to contribute an extra 3% of his salary into the account.

(a) (i) Show that $\$1120$ is deposited in Yusuf's superannuation account every quarter.

(2 marks)

(ii) Calculate the balance of Yusuf's superannuation account when he is 65 years old.

(2 marks)

(iii) Calculate the amount of interest that Yusuf's superannuation account will have earned by the time he is 65.

(1 mark)

At the age of 65, Yusuf intends to deposit $\$1\,090\,000$ into an annuity account that pays an interest rate of 3.5% per annum, compounded weekly. The account will pay him a weekly income for 20 years, after which time the balance will be zero.

(b) (i) If the annuity account conditions remain the same over the 20 years, show that the weekly payment Yusuf can expect is approximately $\$1460$.

(2 marks)

(ii) Calculate the balance in the annuity account after 10 years.

(1 mark)

(iii) Explain why, after 10 years (halfway through the expected term of the annuity), more than half of the original deposit is still in the account.

(2 marks)

(c) Currently, Yusuf is 23 years old and lives on $\$500$ per week.

Assuming that the rate of inflation is 2.5% per annum, use one or more calculations to decide whether Yusuf could maintain his current standard of living *throughout* the term of the annuity.

(2 marks)

Question 3 (6 marks)

An upcoming gymnastics competition has four events (vault, uneven bars, beam, and floor). A club can enter a team of four gymnasts, each of whom competes in a different event. The winning team is determined by the total of its scores in the four events.

A coach has selected the four gymnasts from her club to make a team, but now needs to assign each gymnast to an event. The coach has the placings of the four gymnasts at a recent competition. The table below shows their placings, where 1 represents first place, etc.

Gymnast	*Vault*	*Uneven bars*	*Beam*	*Floor*
Talia	1	6	6	9
Nozomi	5	1	7	1
Kate	9	5	5	6
Martina	6	4	4	6

(a) Use the Hungarian algorithm to find the optimum assignment of the four gymnasts to events at the upcoming competition, based on their placings at the recent competition.

Show that there are *two* solutions and state what they are in the spaces provided.

	Solution 1	Solution 2
Talia	_____	_____
Nozomi	_____	_____
Kate	_____	_____
Martina	_____	_____

(4 marks)

The actual point scores achieved by each gymnast in the events at the recent competition are shown in the table below.

Gymnast	*Vault*	*Uneven bars*	*Beam*	*Floor*
Talia	13.850	12.750	12.125	12.150
Nozomi	13.725	13.675	12.000	13.750
Kate	13.300	13.275	12.300	12.325
Martina	13.625	13.350	12.425	12.325

(b) Use the information in the table to show which solution from part (a) gives the better team for the upcoming competition. *The Hungarian algorithm is* ***not*** *required.*

(2 marks)

Question 4 (10 marks)

Tyler needed a new work van. He found a suitable vehicle for $\$43\,000$ and decided to take out a loan for 6 years. He considered the following options:

Bank	*Nominal rate*	*Establishment fee*	*Charges*	*Comparison rate*
A	6.7% per annum, compounded monthly		$\$0$	6.7% per annum
B	5.69% per annum, compounded monthly	$\$499$	$\$8$ per month	

(a) (i) State why the establishment fee at bank A must be $\$0$.

(1 mark)

(ii) Calculate the comparison rate for bank B.

(3 marks)

Tyler also investigated other banks.

Bank C approved a 6-year loan for Tyler, with an interest rate of 6.3% per annum, compounding monthly. His monthly repayment for this loan is $\$718.74$.

Tyler negotiated with bank C to halve the repayment, and pay $\$359.37$ fortnightly (with interest compounding fortnightly).

(b) (i) Explain *one* disadvantage of Tyler's strategy to make these fortnightly repayments.

(2 marks)

(ii) How many fortnights will it take for Tyler to pay off the loan?

(2 marks)

(iii) Calculate the interest that Tyler will save over the life of the loan by making these fortnightly repayments instead of monthly repayments.

(1 mark)

(iv) State *one other* strategy that Tyler could use to pay off his loan more quickly.

(1 mark)

Question 5 (11 marks)

Organisers of a major international cricket competition in New Zealand are considering in which city each visiting international team should play the majority of its games. To determine this, they are using the number of registered supporters of each team in each city to predict attendances at the games.

The table below shows the approximate number of registered supporters (in hundreds) for each team in five major cities in New Zealand.

Team	*Auckland*	*Christchurch*	*Dunedin*	*Wellington*	*Napier*
Australia	105	80	75	110	60
England	85	75	30	90	65
India	45	30	45	75	70
South Africa	30	28	22	38	48

(a) Explain the meaning of the value 22 in the bottom row of the table.

(2 marks)

(b) Complete the array below to carry out the *first two* steps needed to perform the Hungarian algorithm for this maximisation problem.

5	30	35		50
25	35	80		45
65	80	65		40
80	82	88		62
0				

(2 marks)

(c) Further steps have been done using the Hungarian algorithm to give the array below.

On this array, show that the optimum solution has not yet been reached.

0	25	30	0	45
0	10	55	0	20
25	40	25	0	0
18	20	26	15	0
0	0	0	5	0

(1 mark)

(d) (i) Complete the remaining step(s) of the Hungarian algorithm for the array in part (c).

(2 marks)

(ii) Hence find the optimum solution and interpret it in the context of the problem.

(2 marks)

(e) Explain why the organisers of the cricket competition might not use the solution to this problem provided by the Hungarian algorithm.

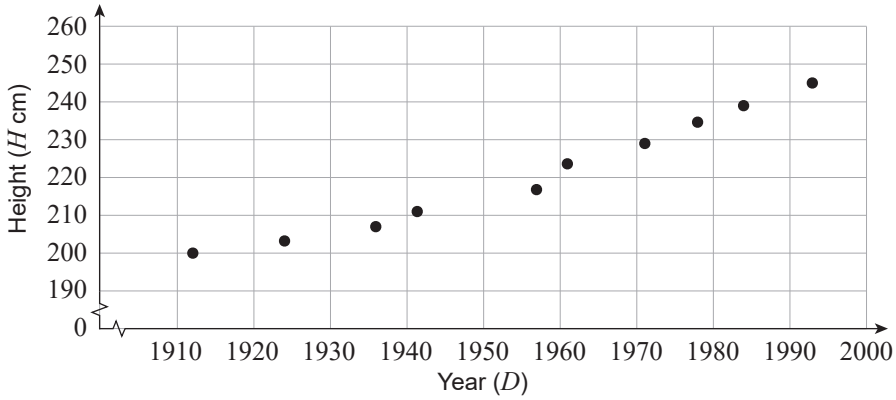
(2 marks)

Question 6 (9 marks)

The table below shows the world record height (H) for the men's high jump (in centimetres), and the year in which it was set (D). The same data are graphed below the table.

Year (D)	1912	1924	1936	1941	1957	1961	1971	1978	1984	1993
Height (H cm)	200	203	207	211	216	223	229	234	239	245

Men's high jump, world record height (1912–1993)



Source: selected data from World Athletics, 'World record progression of high jump', *World Athletics*, viewed 20 July 2020, www.worldathletics.org

(a) (i) Using a linear model, calculate the r^2 value for the relationship between H and D.

(1 mark)

(ii) Hence state the nature and strength of the linear relationship between H and D.

(2 marks)

(b) Which *one* of the following best represents the equation of the linear regression linking height and year? Tick the appropriate box to indicate your answer.

$H = 907D - 0.577$ ☐ $H = 0.577D - 907$ ☐ $H = 1.32 \times 1.003^D$ ☐ (1 mark)

(c) (i) Using your equation from part (b), estimate in what year it predicts a 260 cm high-jump record could be set.

(1 mark)

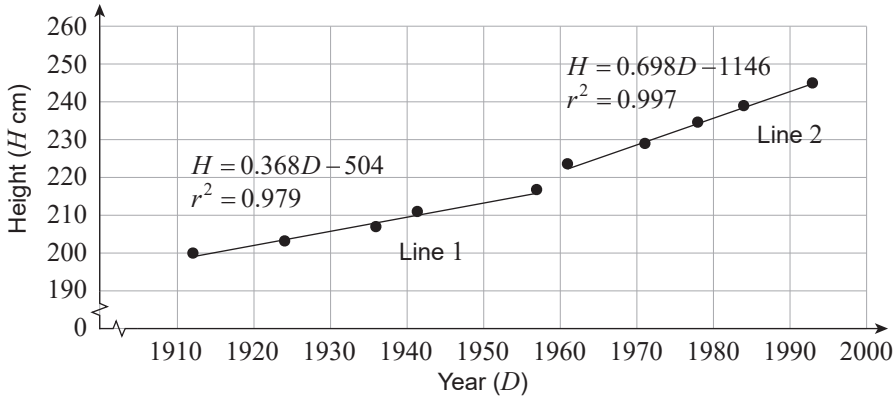
(ii) No new world record has been set for the men's high jump since 1993.

Discuss the reasonableness of your prediction in part (c)(i), by considering your answers to earlier parts of the question.

(2 marks)

In the late 1950s and early 1960s, there were significant changes in high-jumping techniques and the materials used for run-up tracks. Lines of best fit for the data before and after 1960 are shown on the graph below.

Men's high jump, world record height (1912–1993)



(d) Discuss the change in the trend of the data before and after 1960, using interpretation of the slope values for the two lines.

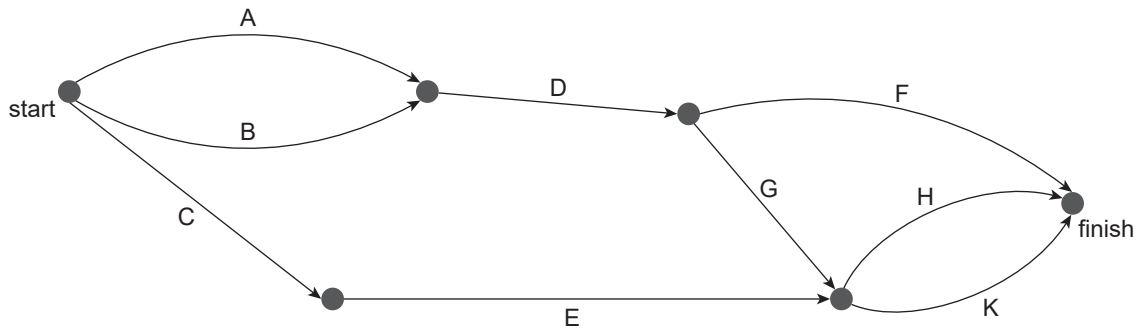
(2 marks)

PLEASE TURN OVER

Question 7 (13 marks)

The precedence table below shows the tasks required to complete a project in a pharmaceutical factory, with a network diagram also shown.

Task	A	B	C	D	E	F	G	H	K
Prerequisites	–	–	–	A, B	A, B, C	D	D	E, G	E, G



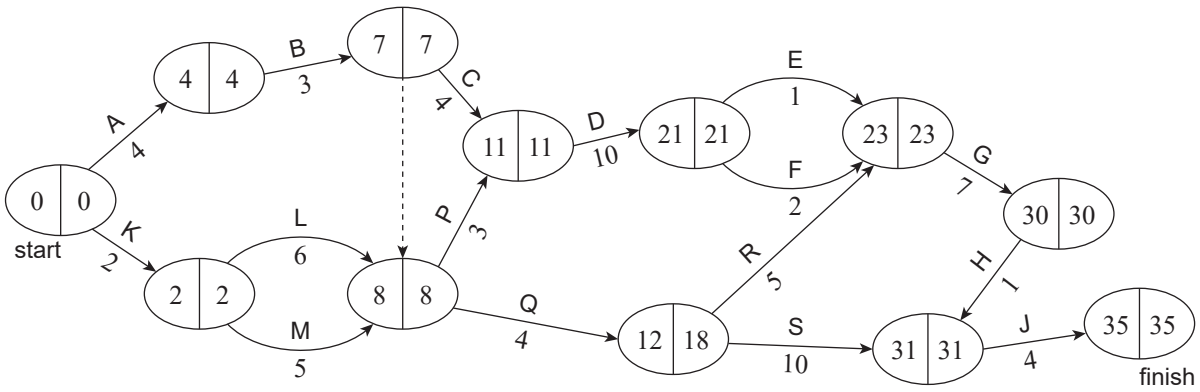
(a) On the network above, ***draw*** the missing dummy link. (2 marks)

(b) Which *one* of the following statements is always true? Tick the appropriate box to indicate your answer.

- [ ] If task H takes more time than task K, then task H must be on the critical path.
- [ ] Tasks H and K cannot both be on the critical path.
- [ ] If task K takes more time than task H, then task H cannot be on the critical path.

(1 mark)

Vitamin C tablets are produced in another part of the factory. The network of tasks for this process is shown below with a forward and backward scan completed. Numbers on the network diagram are times in hours.



(c) State the shortest time in which it is possible to produce the vitamin C tablets.

(1 mark)

(d) List all the tasks that must be completed before task R can be started.

(1 mark)

(e) State the critical path(s) for the production of vitamin C tablets.

(2 marks)

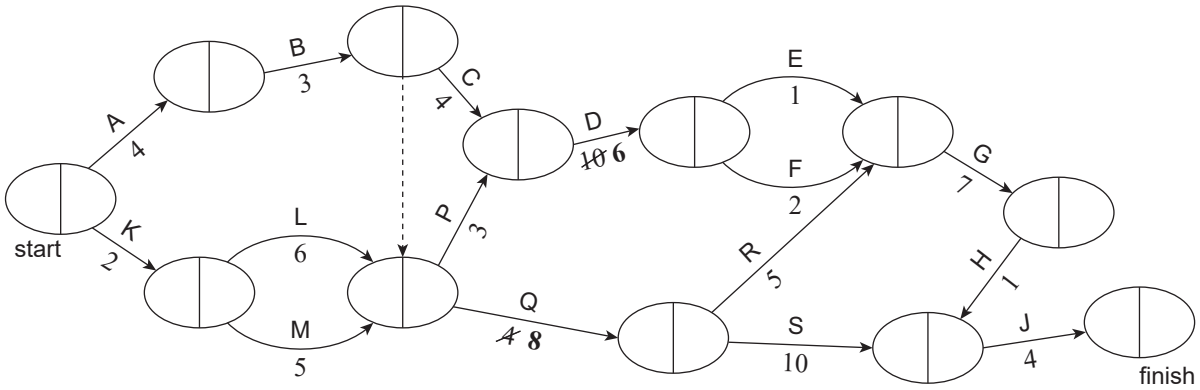
(f) Calculate the maximum time by which task S can be delayed without increasing the minimum completion time.

(1 mark)

PLEASE TURN OVER

The managers reorganise work in the factory and the time for task D is reduced by 4 hours. However, this increases the time taken for task Q by 4 hours.

(g) (i) Carry out a forward and backward scan on the new network diagram below. (2 marks)



(ii) On the network diagram above, ***mark*** the critical path(s). (1 mark)

(iii) The managers had expected the changes to save 4 hours on the minimum completion time.

(1) How much time is actually saved when the changes are made?

(1 mark)

(2) Give a reason why the managers did not save as much time as they expected.

(1 mark)

Question 8 (8 marks)

Maya received an inheritance of \$4200 on her 18th birthday. She invested the money in a savings account that pays interest at a rate of 2.1% per annum, compounded monthly. At this time she also began depositing \$120 into the same account every month.

(a) How old will Maya be when the balance in the account reaches \$10 000?

(3 marks)

(b) (i) Show that Maya will have nearly \$21 200 in her account when she turns 28.

(1 mark)

(ii) Find the amount of interest that will be added to Maya's account over the following year (the year in which she is 28 years old).

(2 marks)

(iii) If Maya's marginal tax rate at age 28 is 37.5%, calculate the amount of tax that she would be required to pay on her account for that year.

(1 mark)

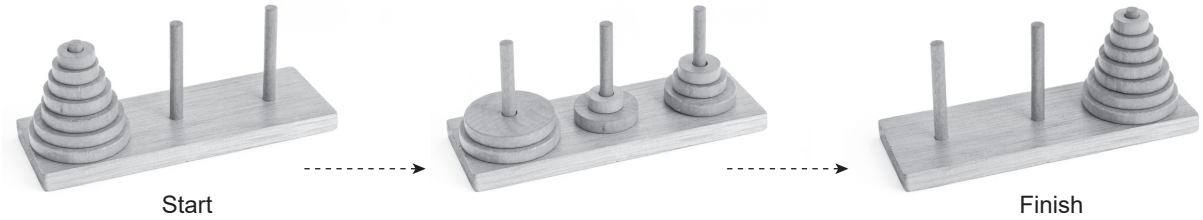
(c) The calculations in parts (a) and (b) assume that the interest rate remains the same.

State another assumption that has been made in these calculations.

(1 mark)

Question 9 (10 marks)

Georgia has been investigating a mathematical puzzle called the Tower of Hanoi which involves moving a set of discs, one disc at a time, from one peg to another according to strict rules.



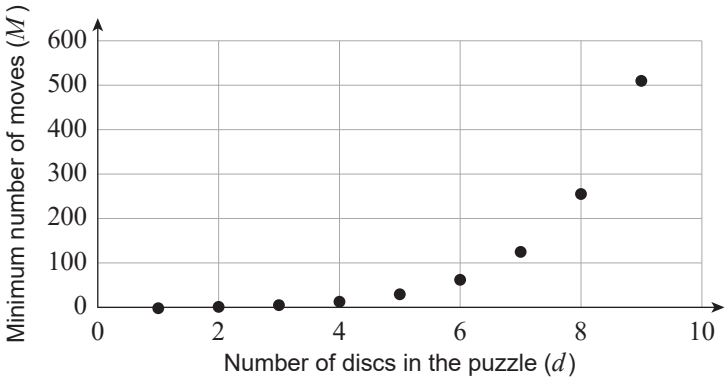
Source: adapted from © dmitryelagin | iStockphoto.com

The minimum number of moves (M) required to solve the puzzle with d discs is displayed in the table below.

Number of discs in the puzzle (d)	1	2	3	4	5	6	7	8	9
Minimum number of moves to solve the puzzle (M)	1	3	7	15	31	63	127	255	511

A scatter plot of the data from the table above is shown below.

Solving the disc puzzle



(a) Based on the scatter plot, state why a linear model would not be appropriate for modelling the relationship in the data above.

(1 mark)

(b) Georgia applies an exponential regression on the data and finds the values $a = 0.629$ and $b = 2.135$.

(i) Using *appropriate variables*, write the equation for the exponential regression in the form $y = a.b^x$.

(1 mark)

(ii) Interpret the value of b in the equation in part (b)(i) in the context of the problem.

(2 marks)

(iii) Find the r^2 value for the exponential model.

(1 mark)

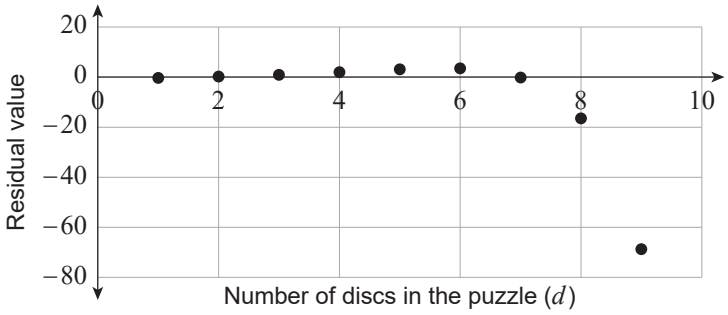
(iv) Using the exponential equation found in part (b)(i), calculate the predicted minimum number of moves needed to solve the puzzle if it has 4 discs.

(1 mark)

Question 9 continues on page 20.

The residual plot below was created for the exponential model.

Residuals for the exponential model



(c) Considering the residual plot, which statement below *best* fits the exponential model? Tick the appropriate box to indicate your answer.

- [ ] The model is a good predictor because most of the residual values are very small.
- [ ] Initially the model is a reasonable predictor, but then it underestimates the number of moves.
- [ ] Initially the model is a reasonable predictor, but then it overestimates the number of moves.
- [ ] The model is a not a good predictor because there is a pattern in the residual plot.

(1 mark)

As part of her investigation, Georgia tries adding 1 to each value of M in her original table to produce the new table below.

Number of discs in the puzzle (d)	1	2	3	4	5	6	7	8	9
Minimum number of moves to solve the puzzle with 1 added (P)	2	4	8	16	32	64	128	256	512

(d) (i) Write down the new exponential regression equation relating P and d for these data, and comment on the significance of the r^2 value.

(2 marks)

(ii) Hence give the equation for the exact relationship between the *original* variables M and d.

(1 mark)

You may write on this page if you need more space to finish your answers. Make sure to label each answer carefully (e.g. 6(c)(i) continued).