## General Mathematics

## Question booklet

- Questions 1 to 9 (90 marks)
- Answer all questions
- Write your answers in this question booklet
- You may write on page 21 if you need more space


## Examination information

## Materials

- Question booklet
- SACE registration number label


## Instructions

- Show appropriate working and steps of logic in this question booklet
- Use black or blue pen
- You may use a sharp dark pencil for diagrams and graphical representations
- Approved calculators may be used - complete the box below

Total time: 130 minutes
Total marks: 90
Attach your SACE registration number label here

| Graphics calculator |
| :---: |
| 1. Brand |
| Model |
| 2. Brand |
| Model |

## Question 1 (9 marks)

Arjun has $\$ 20000$ that he plans to invest for 3 years.
He considers the following two savings accounts:

- Account A - 2.93\% per annum flat rate.
- Account B - 2.82\% per annum, compounding quarterly.
(a) State the effective rate of interest for each account correct to two decimal places.

Account $\mathrm{A}=$ $\qquad$ Account $\mathrm{B}=$ $\qquad$
(b) Arjun decides that he will invest his $\$ 20000$ in a third savings account (Account C) that earns $3.1 \%$ per annum, compounded weekly. He will also deposit $\$ 50$ into the account each week.
Show that the balance of the account will be approximately $\$ 30100$ after 3 years.

(2 marks)

At the start of the investment period, Arjun sees a car valued at $\$ 29000$. He expects that inflation will average $2.5 \%$ per annum over the next 3 years.
(c) (i) Calculate the indexed value of the car in 3 years' time.

(ii) Taking inflation into account, how much extra will Arjun need to contribute to Account $C$ each week in order to have enough money to buy the car in 3 years' time?

(2 marks)
(iii) State one factor, other than the effects of inflation, that Arjun must consider when implementing this savings plan, and explain its impact in the context of this question.

(2 marks)

## Question 2

Company X proposes a contract to renovate an aged-care home. The company submits a network diagram as part of its proposal. The times needed (in weeks) to complete each task, and the order for completion, are shown in the network diagram below.

(a) On the network diagram above, carry out a forward and backward scan.
(b) State the critical path(s) for the network above.

(c) The company claims that allocating extra resources to task $G$ will reduce the minimum completion time for the project.

State why this claim is incorrect.

(1 mark)
(d) Calculate the slack time for task $E$.

(1 mark)
(e) List all tasks that must be completed before task J can be started.


Company $Y$ presents an alternative proposal to renovate the aged-care home. Its network diagram is shown below. Times for completing tasks are given in weeks.

(f) (i) State how long it takes to complete task P.

(ii) State how long it takes to complete task Q .

(iii) State the latest starting time for task S .


By allocating extra resources to one task, it is possible to reduce the minimum completion time of Company Y's proposal to be the same as the minimum completion time for Company X's proposal.
(g) (i) State the task and the reduction in task time required to make this possible.

(ii) Justify your answer to part (g)(i).

(1 mark)

## Question 3 (11 marks)

A company manufactures scented candles. The burning time of vanilla-scented candles is normally distributed with a mean of 252 minutes and a standard deviation of 20 minutes.

The company advertises that they sell vanilla-scented candles with a burning time of at least 238 minutes.
(a) (i) Calculate the probability that a vanilla-scented candle will have a burning time of less than 238 minutes.

(ii) On one day, the company makes 5000 vanilla-scented candles.

Calculate how many of these candles would be expected to have a burning time of less than 238 minutes.

(1 mark)
(b) The company expects to receive complaints from customers who have purchased candles with the shortest $10 \%$ burning time.
Calculate the maximum burning time of a vanilla-scented candle purchased by such a customer.

(2 marks)

The company also manufactures rose-scented candles with a burning time that is normally distributed with a mean of 245 minutes and a standard deviation of 25 minutes.

The company claims that a greater proportion of rose-scented candles will burn longer than 300 minutes, in comparison to the vanilla-scented candles.
(c) Is the company's claim accurate? Support your answer using appropriate calculations and reasoning.


A third variety of candles is manufactured by the company. Lavender-scented candles have a burning time which is normally distributed with a mean of 245 minutes and a standard deviation of 20 minutes.

The diagram below shows the probability distributions for the burning time of the vanilla, lavender, and rose-scented candles.

(d) (i) Using the information provided throughout this question, identify the probability distribution for each variety of candle. On the diagram above, fill in boxes $A, B$, and $C$ to indicate your answers.
(ii) Explain why the maximum height of probability distribution A is lower than both probability distributions B and C, when considering the properties of a probability distribution curve.

(2 marks)

## Question 4 (14 marks)

The table below shows the length of time, in minutes, that it takes four children in the same family to complete weekly chores for the household.

|  | Weekly chores |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Ironing | Folding laundry | Vacuuming | Dusting |
| Kelly | 23 | 17 | 20 | 16 |
| Paula | 27 | 21 | 24 | 16 |
| Bobby | 25 | 19 | 21 | 18 |
| Joe | 25 | 18 | 22 | 15 |

(a) Which statement below is the correct interpretation of the value 19 in the shaded cell?

Tick the appropriate box to indicate your answer.


Bobby takes 19 days to fold laundry.
$\square$
19 pieces of laundry were folded by Bobby.
$\square$
Folding laundry takes Bobby 19 minutes.
$\square$
Bobby delegates folding laundry to Kelly 19 times.
(b) Based on the information in the table above, which child is the fastest to complete the dusting as a weekly chore?

(c) Calculate the total time it would take Kelly to complete all four weekly chores.

(d) (i) Apply the Hungarian algorithm to the array from page 8, given below, to minimise the total time spent on these weekly chores. Show the result of each step clearly.

(ii) Fill in the spaces below to complete the interpretation of your solution in part (d)(i).

|  | Weekly chore |
| :--- | :--- |
| Kelly |  |
| Paula |  |
| Bobby |  |
| Joe |  |

Minimum total time
(2 marks)

The children's father offers to do one of the weekly chores. As a result of this change, a new initial array has been produced, as shown below.

|  | Weekly chores |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Ironing | Folding laundry | Vacuuming | Dusting |
| Kelly | 23 | 17 | 20 | 16 |
| Paula | 27 | 21 | 24 | 16 |
| Bobby | 25 | 19 | 21 | 18 |
| Joe | 25 | 18 | 22 | 15 |
| Children's father | 20 | 16 | 17 | 12 |

(e) State why a dummy column of zeros must be added to the right of the dusting column in order for the Hungarian algorithm to be applied.


The Hungarian algorithm has been applied and the array shown below is reached.

| 1 | 0 | 1 | 2 | 2 |
| :--- | :--- | :--- | :--- | :--- |
| 3 | 2 | 3 | 0 | 0 |
| 1 | 0 | 0 | 2 | 0 |
| 2 | 0 | 2 | 0 | 1 |
| 0 | 1 | 0 | 0 | 4 |

(f) (i) State the member of the household who will not be allocated a weekly chore.

(1 mark)
(ii) State two limitations of using the Hungarian algorithm to allocate weekly chores in the context of this question.

(2 marks)

## Question 5 (8 marks)

Keisha took out a home loan of $\$ 650000$ for a term of 20 years. The loan had an interest rate of $2.88 \%$ per annum, compounded monthly.
(a) (i) Show that the minimum monthly repayment was approximately $\$ 3570$.

(ii) Calculate the total interest that would be charged over the 20 years.


After 5 years, Keisha's balance has reduced to $\$ 520700$ and the loan interest rate increases to $3.6 \%$ per annum, compounded monthly.
(b) (i) If Keisha maintains her repayments from part (a)(i), calculate how much longer it will take her to repay the loan compared to the original loan term.

(ii) However, at this time, Keisha deposits a lump sum payment into the loan.

Calculate the size of the lump sum payment that will be needed at this time in order to repay the loan in the remaining 15 years.


## Question 6 (10 marks)

The table below shows observations of the area of Arctic sea ice from 1980 to 2020.

| Year of observation (D) | 1980 | 1985 | 1990 | 1995 | 2000 | 2005 | 2010 | 2015 | 2020 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Area of Arctic sea ice (S) <br> $\left(\right.$ million $\mathrm{km}^{2}$ ) | 7.54 | 6.49 | 6.04 | 6.03 | 5.76 | 5.32 | 4.62 | 4.43 | 3.82 |

(a) Using correct variables, state the equation for the least squares regression line (line of best fit, $y=a x+b)$ for the linear model.

(2 marks)
(b) Complete the statements below.

The numerical value of Pearson's correlation coefficient $(r)$ for the linear regression model is $\qquad$ .

This suggests that as the $\qquad$ increases, the $\qquad$ will $\qquad$ .

The table below gives the residual values for the linear model.

| Year of observation (D) | 1980 | 1985 | 1990 | 1995 | 2000 | 2005 | 2010 | 2015 | 2020 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Linear model residual <br> (rounded to three <br> decimal places) | 0.338 | -0.301 | -0.341 | 0.059 | A | 0.169 | -0.120 | 0.099 | -0.100 |

The residual plot for the linear model is shown below.

(c) One residual value is listed in the second table on page 12 as $\mathbf{A}$.
(i) Calculate the value of $\mathbf{A}$.

(ii) On the residual plot above, plot the point $\mathbf{A}$.
(d) (i) Using your equation from part (a), predict the year of observation when the area of Arctic sea ice will be 1 million $\mathrm{km}^{2}$.

(1 mark)
(ii) Discuss the reliability of the prediction made in part (d)(i). Give two reasons for your answer.

(2 marks)

## Question 7

The Flamingo Mask Company has identified nine activities that take place when manufacturing its masks. These are listed in the precedence table below.

| Activity | Task time <br> (hours) | Immediate <br> predecessors | Earliest <br> starting time <br> (hours) | Latest <br> starting time <br> (hours) |
| :---: | :---: | :---: | :---: | :---: |
| A | 5 | - | 0 | 1 |
| B | 6 | - | 0 | 0 |
| C | 4 | A, B | 6 | 11 |
| D | 3 | B | 6 | 9 |
| E | 6 | B | 6 | 6 |
| F | 3 | C | 10 | 16 |
| G | 3 | D | 9 | 14 |
| H | 5 | D, E | 12 | 12 |
| J | 2 | F, G, H | 17 | 17 |

The following network diagram has been produced from the information in the precedence table above.

(a) On the network diagram above, fill in the boxes to indicate where tasks D and E should be placed. A forward and backward scan are not required.
(b) Explain why the dummy link marked X is required.

(c) Which option below correctly identifies the critical path and minimum completion time for this project? Tick the appropriate box to indicate your answer.

Critical path Minimum completion time
$\square$ $B, E, H, J \quad 17$ hours
$\square$ $B, D, H, J \quad 16$ hours$B, D, H, J$
19 hours$B, E, H, J$
19 hours
(d) Which of the following statements are correct? Tick the appropriate box(es) to indicate your answer(s).
$\square$ Task J will always be on the critical path because it takes the shortest time to complete.Task A has no slack time.
$\square$ Task J will always be on the critical path because it is the only task leading to the end of the network.Dummy link $Y$ takes more time to complete than dummy link $X$.A delay in the completion of task J will always increase the minimum completion time for the project.

## Question 8

Olivia is interested in purchasing a small apartment for $\$ 280000$. She wants to borrow $\$ 280000$ over 15 years. Her financial adviser suggested the following options:

- Option A - 3.89\% per annum, compounding fortnightly, with an establishment fee of $\$ 1500$ and an ongoing service fee of $\$ 15$ per fortnight.
- Option B - 4.25\% per annum, compounding fortnightly, with no additional charges.
(a) Calculate the comparison rate for Option A.

(3 marks)
(b) Which statement below most accurately describes the best option for Olivia? Tick the appropriate box to indicate your answer.Option B is best as there are no additional charges.
$\square$
Option A is best as $3.89 \%$ per annum is lower than the $4.25 \%$ per annum for Option B.
$\square$
Option A is best as it has the lowest comparison rate.
$\square$ Option B is best as it has the highest comparison rate.
$\square$ Option B is best as it has the lowest comparison rate.

Olivia chooses a different option where she will pay a deposit of $\$ 50000$ and take out a loan for the remaining balance. Her weekly repayments will be $\$ 396$ for 15 years.
(c) Show that the interest rate charged on this loan is approximately $4.1 \%$ per annum, compounding weekly.

(2 marks)
(d) Calculate the time it will take for the balance of the loan to reach approximately half of its principal value (\$115000).

(1 mark)

When the balance of the loan has reduced to $\$ 115000$, Olivia receives an inheritance that would allow her to pay off the loan in full. Her financial adviser presents her with the following options:

- Option X — Repay the loan in full immediately, with an additional early repayment charge of $11 \%$ of the current balance.
- Option Y — Refinance the loan for 6 years with interest charged weekly at $3.5 \%$ per annum, compounding weekly, with no additional charges.
(e) (i) Calculate the cost of the early repayment charge if Olivia chooses Option X .

(ii) Calculate the total interest that Olivia will be charged if she chooses Option Y.

(2 marks)
(iii) Olivia chooses Option $Y$ and deposits $\$ 115000$ into an offset account linked to Option Y.

State two reasons to explain why this is a reasonable decision.

(2 marks)

## Question 9 (9 marks)

After medication V is administered into the bloodstream, the concentration of the medication present decreases over time.

The table below shows the concentration of medication V in the bloodstream (in $\mu \mathrm{g} / \mathrm{mL}$ ) at time $t$ hours after the medication was administered.

| Time after medication V is <br> administered (t hours) | 0 | 2 | 5 | 7 | 10 | 12 | 15 | 18 | 20 | 25 | 30 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Concentration of medication V <br> in the bloodstream $(C \mu \mathrm{~g} / \mathrm{mL})$ | 1000 | 765 | 550 | 418 | 316 | 222 | 180 | 100 | 85 | 56 | 31 |

A regression equation, coefficient of determination, and residual plot for the linear model are given below.

| Linear model |
| :---: |
| $C=-29.9 t+729$ |
| $r^{2}=0.807$ |

Residual plot for linear model

(a) Using the information above, explain why the linear model is not appropriate to use for making predictions from this data.


The equation for the exponential model, in the form $y=a \cdot b^{x}$, was found to be:

$$
C=962 \times(0.89)^{t} .
$$

(b) (i) Which statement below provides the complete interpretation of the value $b$ in the context of the question? Tick the appropriate box to indicate your answer.The concentration of medication V in the bloodstream increases by $89 \%$ per hour.The concentration of medication V in the bloodstream increases by $11 \%$ per hour.The concentration of medication V in the bloodstream decreases by $89 \%$ per hour.
$\square$ The concentration of medication V in the bloodstream decreases by $11 \%$ per hour.
(ii) State why the value of $a$ in the exponential model is an underestimate of the original data.

(c) Using the exponential model $C=962 \times(0.89)^{t}$,
(i) calculate the concentration of medication V in the bloodstream 24 hours after the medication was administered.

(ii) calculate the time taken for the concentration of medication V in the bloodstream to reduce to $481 \mu \mathrm{~g} / \mathrm{mL}$.

(1 mark)

A rival company produces a generic version of the medication, known as medication W.
The equation for the exponential model of the concentration of medication W in the bloodstream $(C)$ is:

$$
C=970 \times(0.75)^{t}
$$

where $t$ is the number of hours after administration of the medicine.
(d) (i) Calculate the time taken for the concentration of medication W in the bloodstream to reduce to $481 \mu \mathrm{~g} / \mathrm{mL}$.

(ii) With reference to the parameters of the model, explain why there is a difference in the time taken for the concentration of medication $W$ in the bloodstream to reduce to $481 \mu \mathrm{~g} / \mathrm{mL}$, compared with medication V.

(2 marks)

You may write on this page if you need more space to finish your answers. Make sure to label each answer carefully (e.g. 9(b)(ii) continued).
$\qquad$

