Government of South Australia

SACE Board of SA | South Australian Certificate of Education

General Mathematics
2017

Question Booklet

- Questions 1 to 9
- Answer ***all*** questions
- Write your answers in this question booklet
- You may write on page 22 if you need more space
- Approved calculators may be used — complete the box below

Examination information

Materials

- Question Booklet
- SACE registration number label

Reading time

- 10 minutes
- You may make notes on scribbling paper

Writing time

- 2 hours
- Show all working in this question booklet
- Use black or blue pen
- You may use a sharp dark pencil for diagrams

Total marks 90

Attach your SACE registration number label here

Graphics calculator

1. Brand _____

Model _____

2. Brand _____

Model _____

For office use only

Supervisor check	Re-marked

Question 1 (12 marks)

Carbon monoxide is a poisonous gas emitted by motor vehicles. A local environmental group monitors the concentration of carbon monoxide in the air.

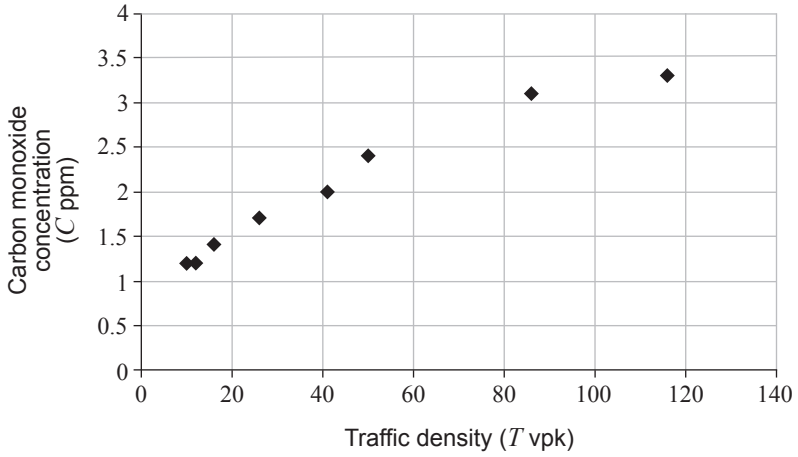
The average summer weekday carbon monoxide concentration, C parts per million (ppm), and the traffic density, T vehicles per kilometre (vpk), is measured over 9 days. The results are shown in the table below.

Day	1	2	3	4	5	6	7	8	9
Traffic density (T vpk)	50	26	16	10	12	41	87	116	120
Carbon monoxide concentration (C ppm)	2.4	1.7	1.4	1.2	1.2	2.0	3.1	3.3	3.5

Source: Adapted from *Carbon Monoxide from a Freeway*, statsci.org/data/general/cofreewy.html

(a) On the scatter plot below, add the data point that corresponds to the last column in the table above.

Carbon monoxide concentration (C) versus traffic density (T)



(1 mark)

(b) (i) State the coefficient of determination (r^2) for a linear regression on the data in the table above.

(1 mark)

(ii) Describe the nature and strength of the linear relationship between the variables.

(2 marks)

(c) (i) State the equation of the least squares regression line in terms of C and T.

(2 marks)

(ii) State the value of 'a' (the slope of the least squares regression line), and interpret this value in the context of the question.

(2 marks)

(d) Using the equation that you stated in part (c)(i) on page 3, calculate:

(i) the carbon monoxide concentration, C, when the traffic density, T, is 250 vpk.

(1 mark)

(ii) the traffic density, T, when the carbon monoxide concentration, C, is 2.5 ppm.

(1 mark)

(e) Which of the two values that you calculated in part (d) should be more reliable? Justify your answer.

(2 marks)

Question 2 (8 marks)

Ezekiel has decided to set up his own health fund in a savings account, instead of taking out private health insurance. He will deposit fortnightly payments of $\$64.50$ into the account, which earns an interest rate of 3.45% per annum, compounded fortnightly.

(a) State *one* factor (other than changes in interest rates) that Ezekiel should consider when providing for his health costs in this way.

(1 mark)

Ezekiel has been told that he might need knee surgery in 5 years' time. This type of surgery currently costs around $\$17\,000$.

(b) (i) Calculate how much Ezekiel would have in his account after 5 years.

(2 marks)

(ii) Ezekiel wants to pay the full cost of the surgery from the money in his savings account.

In order to do this, calculate how much longer he will have to wait for the surgery, after the 5 years.

(2 marks)

(c) Ezekiel was able to wait 12 years for the surgery. The annual inflation rate over the 12 years averaged 2.8% per annum.

Given that 12 years ago the expected cost of the surgery was $\$17\,000$, calculate how much the surgery could cost now.

(1 mark)

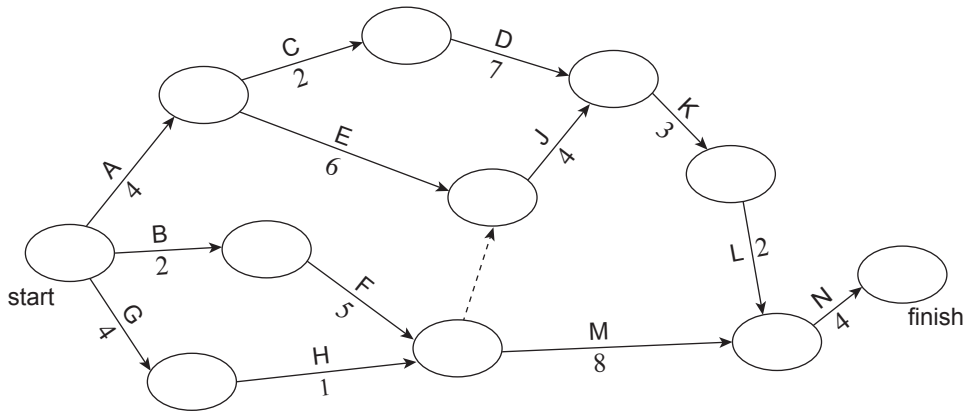
(d) At the age of 65, Ezekiel has a total of $\$97\,800$ left in the health fund savings account. He has now made different arrangements to cover his health costs. He decides to invest this amount into an annuity to pay himself an income. The annuity pays 5.7% interest per annum, compounded weekly.

Calculate the weekly income that Ezekiel can expect if the annuity is to last for 10 years.

(2 marks)

Question 3 (13 marks)

A company uses multiple tasks to make a product. The times needed (in minutes) to complete each task, and the order for completion, are shown on the following network diagram:



The company uses a critical path analysis to identify the minimum completion time to make the product.

(a) Using the network diagram above, fill in the *four* blank cells to complete the precedence table below.

Task	A	B	C	D	E	F	G	H	J	K	L	M	N
Time (minutes)	4	2	2	7	6		4	1	4	3	2		4
Prerequisite task(s)	–	–	A	C	A		–	G	E, F, H	D, J	K		L, M

(2 marks)

(b) (i) Which task is the only one that *must* be on the critical path?

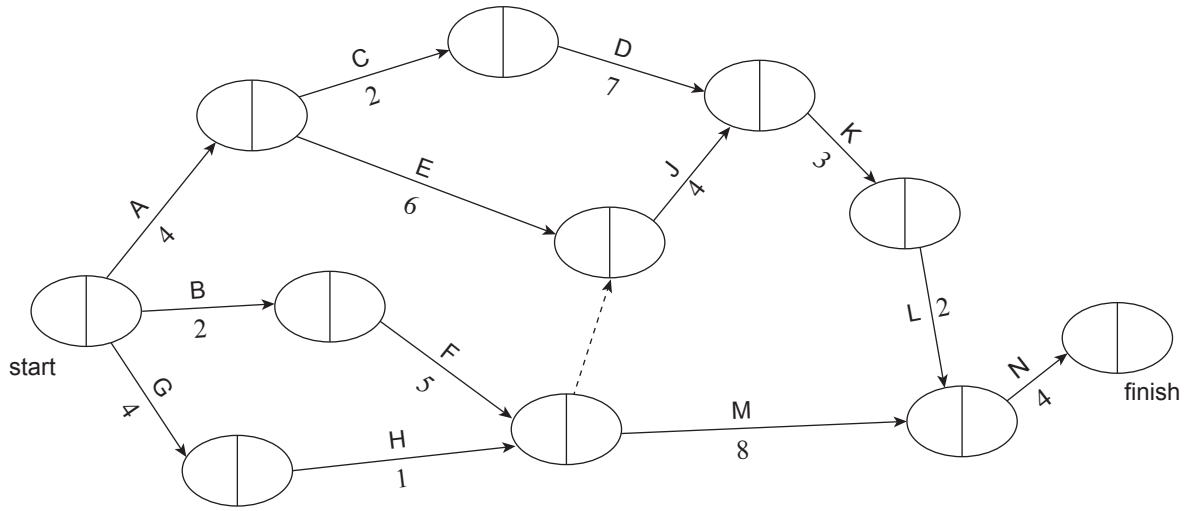
(1 mark)

(ii) Using information in the network diagram above, explain why task G will ***not*** be on the critical path.

(2 marks)

PLEASE TURN OVER

(c) Using the following network diagram:



(i) complete a forward and backward scan. (2 marks)

(ii) state the critical path and minimum completion time to make the product.

(2 marks)

(d) With reference to the network diagram above, explain the purpose of the dummy link (shown as a dotted line).

(2 marks)

(e) Kurt, a trainee employee, takes on average 4 minutes longer than other employees take to complete any task.

State *one* of the tasks Kurt could be given that would not increase the minimum completion time. Use figures from the network diagram on page 8 to justify your answer.

(2 marks)

Question 4 (10 marks)

Light World makes the Super light bulb, which has a mean working life of 5100 hours, with a standard deviation of 200 hours. The working life data for the Super light bulb are normally distributed. Light World advertises that the Super light bulb has a working life of 5000 hours.

(a) Calculate the proportion of Super light bulbs that should have a working life of at least 5000 hours.

(2 marks)

(b) Calculate the probability that a single randomly selected Super light bulb would have a working life of between 4800 and 5000 hours.

(1 mark)

(c) Light World guarantees to replace any Super light bulb that stops working before a certain number of hours.

If the company wants to replace no more than 8% of the Super light bulbs, at how many hours of working life should it set the guarantee?

(2 marks)

Light World develops an improved light bulb that has a mean working life of 5300 hours, with a standard deviation of 100 hours. The working life data for the improved light bulb are normally distributed.

(d) The company makes $122\,550$ of the improved light bulb.

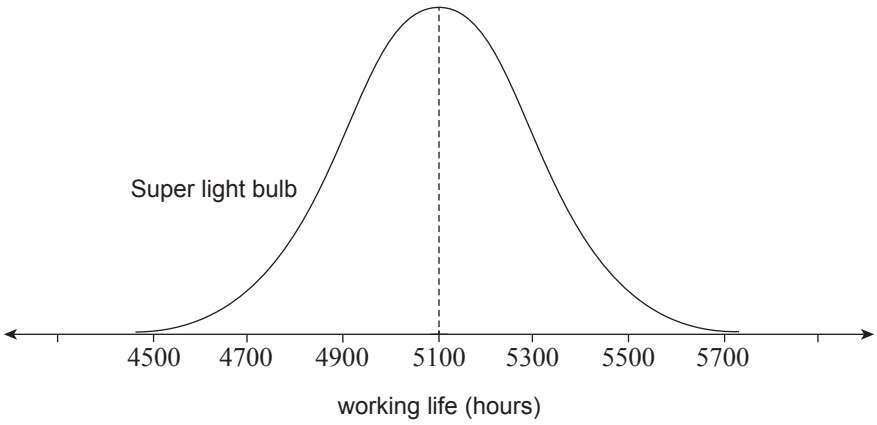
How many of these light bulbs can the company expect to have a working life of at least 5000 hours?

(2 marks)

(e) The following normal distribution graph shows the working life of the Super light bulb.

On the scaled axis below, add a graph to represent the working life distribution of the improved light bulb.

Working life of light bulbs



(3 marks)

Question 5 (10 marks)

Hossam has saved $\$7520$ towards his goal of travelling overseas. He wants to put this amount into a new savings account and continue saving for the next 2 years.

(a) Hossam has identified two possible accounts to choose from:

- Account 1 offers an interest rate of 3.8% per annum, compounded monthly.
- Account 2 offers a flat interest rate of 4% per annum over 2 years.

Which account (1 or 2) would give the better return? Show working to support your answer.

(4 marks)

Hossam decides not to invest in Account 1 or Account 2. Instead he invests the $\$7520$ in Account 3, which pays 4.3% interest, compounded weekly. He makes weekly deposits of $\$40$ into this account.

(b) (i) Show that Hossam will have earned approximately $\$860$ in *interest* in Account 3 after 2 years.

(3 marks)

(ii) Calculate the amount that Hossam will pay in tax on the earnings from Account 3, if his marginal tax rate is 32.5%.

(1 mark)

(iii) State *one* assumption (apart from the interest rate remaining constant) that is being made when doing the calculations in parts (b)(i) and (ii), and discuss its possible effect on the results.

(2 marks)

Question 6 (14 marks)

A small transport company has four trucks (T1, T2, T3, and T4) available to pick up packages from four locations: airport, railway station, ferry terminal, and city centre. The distances (in kilometres) that each truck must travel to get to the locations are shown in the table below.

Truck	*Airport*	*Railway station*	*Ferry terminal*	*City centre*
T1	11	19	17	18
T2	21	15	10	13
T3	15	21	19	21
T4	16	13	20	19

(a) Interpret the meaning of the number '20' in the table above.

(1 mark)

(b) The company wants to assign trucks to the locations to minimise the total distance travelled.

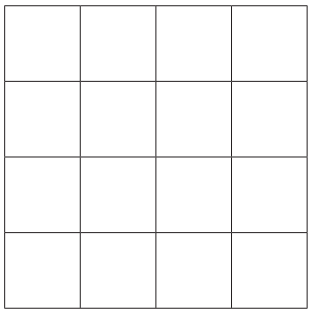
(i) The first two steps of the Hungarian algorithm have been applied, and the resulting reduced array is shown below.

Draw lines on the reduced array below, and explain how you know that the optimal solution has not yet been reached.

0	8	6	4
11	5	0	0
0	6	4	3
3	0	7	3

(2 marks)

(ii) Using the reduced array in part (b)(i) on page 14, complete the application of the Hungarian algorithm below.



(3 marks)

(iii) For the optimal solution, state the assignment of trucks to the locations and the total distance travelled.

(2 marks)

Question 6 continues on page 16.

(c) A fifth truck, T5, has become available, and the distances it has to travel to reach the four locations are shown in the table below.

It has been found that major roadworks will add 4 kilometres to the distance truck T1 has to travel to the airport, and add 6 kilometres to the distance truck T2 has to travel to the ferry terminal.

(i) Fill in the blank cells to complete the table below.

Truck	*Airport*	*Railway station*	*Ferry terminal*	*City centre*
T1		19	17	18
T2	21	15		13
T3	15	21	19	21
T4	16	13	20	19
T5	17	14	16	16

(1 mark)

(ii) Apply the Hungarian algorithm to the array in the table in part (c)(i). Show your working in the space below.

(3 marks)

(iii) Interpret your result in part (c)(ii) in the context of the question.

(2 marks)

Question 7 (4 marks)

Anya makes scented candles for sale. The tasks, A to K, and the prerequisites for each task, are shown in the precedence table below.

Task		*Prerequisites*
A	Chop wax	–
B	Melt wax	A
C	Cut wicks	–
D	Tie wicks to candle moulds	C
E	Add scented oils to melted wax	B
F	Fill moulds	E, D
G	Cut wrappings	F
H	Cut ribbons	F
I	Candle setting (cooling)	F
J	Remove candles from moulds	H, I
K	Package candles	G, J

Using the information in the precedence table above, draw and label (using A to K) a network diagram.

(4 marks)

Question 8 (7 marks)

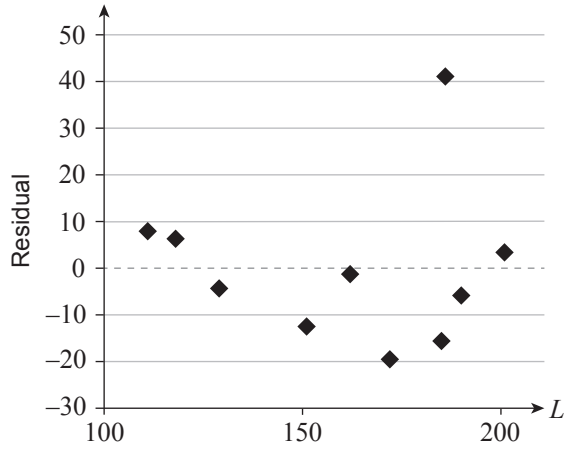
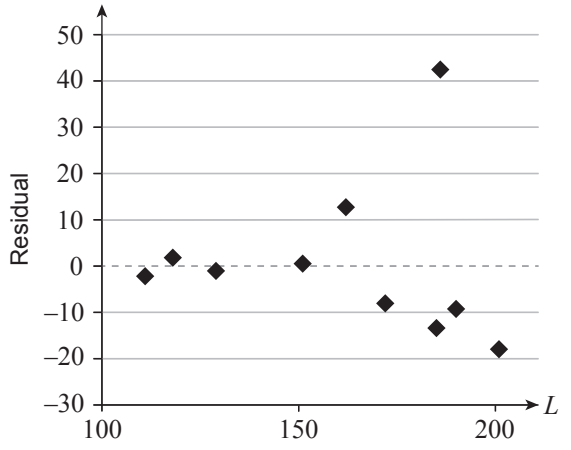
A scientific survey of shark species in the Great Australian Bight recorded the average length (in centimetres) and the average weight (in kilograms) of 10 species of shark. The data are shown in the table below.

Average length (L cm)	111	118	129	151	162	172	185	186	190	201
Average weight (W kg)	15	22	25	44	69	63	83	141	99	122

Source: Adapted from www.nefsc.noaa.gov/publications/tm/tm110/tm110.pdf

Both a linear model and an exponential model were found for the data.

A residual plot, regression equation, and coefficient of determination for both the linear model and the exponential model are given below.

Linear model	**Exponential model**
	
$W = 1.24L - 130.45$	$W = 1.28 \times 1.024^L$
$r^2 = 0.843$	$r^2 = 0.951$

(a) Using the information above, explain why the exponential model is the more appropriate to use for predicting from the data in the table.

(2 marks)

(b) Identify a possible outlier in the data.

(1 mark)

(c) The outlier that you identified in part (b) was found to be a recording error.

Remove the outlier. Write the new exponential regression equation, and recalculate its coefficient of determination (r^2).

(2 marks)

(d) A shark has been caught in the Great Australian Bight, and scientists think it is a new species. The length of this shark is 143 centimetres.

(i) Using this length and the exponential regression equation that you found in part (c), predict the average weight of the new species of shark.

(1 mark)

(ii) State why making the prediction in part (d)(i) may not be reasonable.

(1 mark)

Question 9 (12 marks)

Pia needs to borrow $\$80\,000$ to renovate her house. She is aiming to repay the loan over 7 years and is investigating two loan options:

- Option A is a loan with an interest rate of 6.25% per annum, compounded monthly, with no set-up fee or ongoing fees.
- Option B is a loan with an interest rate of 6.15% per annum, compounded monthly, with a $\$200$ set-up fee and a $\$7$ monthly fee.

(a) Calculate the comparison rate for Option B.

(4 marks)

(b) Complete the statement below:

Option _____ is the loan that represents the better value because _____

_____ . (1 mark)

Only the bank that offers Option A has approved Pia's loan application. She borrows $\$80\,000$ at 6.25% per annum, compounded monthly for 7 years.

(c) Show that her minimum monthly repayment is approximately $\$1180$.

(2 marks)

(d) (i) Show that Pia's outstanding debt 3 months after taking out the loan will be approximately $\$77\,700$.

(1 mark)

At this time, Pia sells some of her furniture for $\$4000$. She deposits this money in an offset account that is attached to her loan account.

(ii) If Pia continues the minimum monthly repayment in part (c), calculate how long it will take her to repay the loan now that she has $\$4000$ in the offset account.

(1 mark)

(iii) Calculate *approximately* how much interest will be saved by using the offset account.

(2 marks)

(iv) State *one* assumption made that might affect the amount of interest saved by using the offset account.

(1 mark)

You may write on this page if you need more space to finish your answers. Make sure to label each answer carefully (e.g. 'Question 6(c)(ii) continued').

GENERAL MATHEMATICS 2017

ACKNOWLEDGMENT

Question 1, table: Data Source: Hogg, R. V., and Ledolter, J. (1992). Applied Statistics for Engineers and Physical Scientists, Second Edition. Macmillan, New York. Exercise 1.5-6

Question 8, table A: Kohler NE, Casey JG, Turner PA, 1996, 'Length-length and length-weight relationships for 13 shark species from the Western North Atlantic', US Dep Commer, NOAA Tech Memo NMFS NE 110; 22 p.